HYDROSTATICS

FOR BEGINNERS

BY

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HYDROSTATICS

FOR BEGINNERS

By the same Author.

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ELECTRICITY AND MAGNETISM

FOR BEGINNERS

MACMILLAN & CO., Ltd., LONDON.

PREFACE.

This book is intended to serve as a class-book of Hydrostatics in schools, and as a text-book for those examinations in which a knowledge of elementary hydrostatics is required.

It is beginning to be generally recognised that Physics should be taught in the laboratory as well as in the class-room; by experiments which the boys themselves perform, as well as by 'lectures,' and 'lecture experiments.' For beginners it is necessary that the experiments should be worked concurrently with the class work, and consequently that all the class should be working the same experiment at the same time. This requires a multiplication of apparatus, but as the apparatus is necessarily simple, it will be found inexpensive, and no difficulty is met with in working classes of thirty boys in this manner. When the student is reading the subject for the second time, he will have reached the stage when more accurate results are aimed at, and

when more sensitive apparatus is required. More advanced classes can then be conducted on the 'separate experiment' system, as each student will be in a position to work intelligently at his own experiment.

To Chaps. II.—V. I have appended easy quantitative experiments, which it is recommended should be performed by the class working in pairs. As the object of these experiments is to enable the student to verify for himself the main principles of the subject, it is important that he should write out a clear account of each experiment *as it proceeds,* so that he may follow exactly the steps he is taking. If accuracy of results were the chief aim, the converse of this method is, as is well known, the best. In the description of the experiments I have as far as possible avoided giving too many details, as I am strongly of opinion that the pupil should be set to work with as few instructions as possible, and that he should be left to make his own deductions, and find by experience the precautions which he must take. The account of the experiments should in all cases be written out in the boy's own words, and not according to any stereotyped plan, however tempting this latter may be, in that it makes the work of a class more uniform and easy (and seductively neat) by reducing it to a rule. For a first course it

will be found necessary to adhere fairly closely to the subjects contained in these experiments, and to take only those parts of the main text which bear upon them. The propositions have been rigorously deduced from the fundamental property of a fluid, but with young beginners it will be advisable to omit some of the deductions and base the results upon independent experiments. With this view I have given experimental verifications of the fundamental propositions.

At Dulwich Physics is taught to all the forms on the Modern side, and in a lower form on the Classical side. The average time given to the subject by such forms is two hours per week. This consists of one hour class work, and one hour in the laboratory. The course is (1) Mensuration, (2) Hydrostatics, (3) Heat, (4) Dynamics, (5) Magnetism, (6) Electricity, and (7) Optics. The lower forms take (1), (2), and (5), and the upper forms (3), (4), (6), and (7), and revise the others. In this way it is found possible to take the boys in forms, and as each boy moves up the school he goes through the complete course. The same course is also taken by boys in the lower forms of the Science and Engineering sides.

The fittings of a laboratory for a course such as this are very simple. All that is required is a large room

with tables about 8 ft. by 4 ft., with a gas jet and four-way tap at the centre. Placed at the sides of the room, and near each table, should be a large deep sink with two taps. At each table eight boys can work, and any water they require can without confusion be taken from the corresponding taps. It is important that there should be plenty of space between the tables. In a room 40 ft. by 30 ft., there is room for five tables with a width of 8 ft. between them, besides leaving room for advanced work to be done at the sides.

I have ventured to restrict the word 'pressure' to mean 'force per unit area,' and have used the word 'thrust' or 'force,' where it has been usual to use 'resultant pressure,' 'whole pressure,' or simply 'pressure.' In the same way I have restricted the word 'tension' to 'force per unit area,' and have used the word 'pull' when force is meant. These distinctions I have found of great value in teaching the subject to boys.

I have also endeavoured to restrict the word 'weight' to the precise meaning of force due to the attraction of the earth, and have used 'mass' for quantity of matter. As the student advances into other parts of physics he meets with a large number of quantities, each quantity being measured in terms of some unit, and nothing tends more to make his ideas clear and precise than a

rigid adherence to a fixed nomenclature, each quantity and each unit being denoted by a distinctive name. In Chapter II. and in a few other parts of the subject, the propositions and examples might equally well be stated in terms either of the weight or of the mass of the bodies under consideration. The word 'weight' is also frequently used as an abbreviation for the phrase 'a body whose weight is,' and also simply for 'a body.' It is, I think, advisable to avoid as far as possible the use of 'weight' in this sense.

The book will be found to contain all of the subject required for those examinations in which a knowledge of Elementary Hydrostatics is required; and to make it more useful in this respect I have added a collection of papers set at various examinations. I have not thought it advisable to alter the nomenclature in these examination papers. The student will be able to determine what is meant from the context.

My thanks are due to Mr. W. N. SHAW, M.A., Fellow and Lecturer of Emmanuel College, for many valuable suggestions, especially as regards the definitions and nomenclature. My thanks are also due to my pupil Mr. D. H. MOORE for the results of the numerous illustrative experiments, which were all worked with the apparatus described, and give an idea of the accuracy

attainable in careful hands; to Mr. R. LL. JONES, B.A., now Professor of Physics in the Madras Presidency College, for kindly working through the majority of the examples; and to Mr. S. H. WELLS, Wh.Sc., Demonstrator in the Mechanical Laboratory, for a large number of drawings which will make the subject intelligible and interesting.

F. W. SANDERSON.

DULWICH COLLEGE.
September, 1889.

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ELEMENTARY HYDROSTATICS.

CHAPTER I.

On the Properties of Matter.

1. The existence of matter is made known to us by our senses. Matter for our present purposes may be defined to be "Whatever occupies space."

A **body** is the name given to a portion of matter.

With regard to the space a body occupies there are two ideas which at once occur:

(i) The **shape** of the body, or the form of the surface bounding the body.

(ii) The **volume** or bulk of the body.

Special well-known names are given to certain well-defined shapes. The shapes which will be met with in elementary hydrostatics are the Cube, Prism, Cylinder, Pyramid, Cone, and Sphere. Exact definitions of these shapes are given in books on Geometry and Mensuration.

The volume of a body is the space it occupies, and it is expressed in some unit of volume. The unit of volume depends upon the unit of length, and the unit

of length is generally chosen so as to be most convenient for the particular purpose. In measuring the distance between two towns, the mile is taken as the unit of length in England; in surveying, the chain is taken as the unit of length; in building, the foot is taken as the unit of length.

All these units are multiples or sub-multiples of the British Standard Unit of Length, viz. the *yard*. The **yard** is defined as the distance between the centres of two gold plugs in the bronze bar deposited in the office of the Board of Trade when the temperature of the bar is 62° F. The **foot** is one-third of this distance. The relations of other units of length to the standard are given in Table I.

The disadvantage of the English system of units is that they are not connected by convenient ratios. In the Metric system of units the standard of length is the *Metre*, and all the other units are decimal multiples or sub-multiples of the metre. The metre was supposed to represent the 10,000,000th part of a quadrantal arc of a meridian on the earth's surface. But more refined measurements may prove that the earth's quadrantal meridian is not exactly 10,000,000 metres. The *Metric standard* **metre** *was made of platinum by Borda, and is the distance between the centres of two gold plugs in the bar of platinum when the temperature is* 0° C. The **centimetre** is 100th of a metre, and is the Metric unit which will be adopted in this book. The relations of the units of length are given in Table II. The metre is about 39·37 inches, *i.e.* about 3¼ feet.

The unit of area is the area of a square, the length of a side of which is the unit length.

The unit of volume is the volume of a cube, the length of the edge of which is the unit length.

If 1 foot is the unit of length, the volume of a cube, the edge of which is one foot, is the unit of volume, and is called a cubic foot.

If 1 centimetre is the unit of length, the volume of a cube, the edge of which is one centimetre, is the unit of volume, and is called a cubic centimetre.

The method of finding the areas of surfaces and the volumes of bodies from their linear dimensions is part of the subject of mensuration. The areas of certain well-known plane figures and the volumes of well-known solids are given in Table III.

2. **Elastic and Plastic Bodies.** If force be applied to a body it will either alter its volume, its shape, or both its volume and shape. If the volume is altered, the particles of the body are either brought nearer together or are driven further apart. If the shape of a body is altered, then one part of the body is made to slide over the other. In both cases a displacement of *the particles takes place, and the body is said to be* **strained.** The following experiments will show the different behaviour of bodies when acted upon by force:

(1) Take a piece of india rubber about a foot long and a quarter of an inch thick, and fasten one end to a fixed support, and suspend weights at the other end. The india rubber will be found to extend considerably for moderate forces until its length is about eight times

its original length. When the weights are taken off the india rubber will nearly return to its initial state.

(2) Compress a piece of cork by placing weights on it, and notice when the weights are taken off that the cork resumes its original form.

(3) Suspend a fine steel wire, and hang weights from the lower end. It will be found to require considerable force to alter its length by even a small amount. (The small increase of length can be magnified by a light lever.) When the weights are taken off the wire will return almost exactly to its original length.

(4) Perform the same experiment, using a length of copper wire. It will be found to stretch easily with moderate weights, and on the weights being taken off it will not return to its original form; the wire will exert very little force of recovery.

(5) Flatten a lead bullet by hammering. The lead will remain in the state in which it is hammered; it will exert no force of recovery.

(6) A piece of putty can easily be moulded into any form.

India rubber, cork, and steel were found to exert (within the limits of the experiments) forces tending to restore their shape or volume, so that when the stretching or compressing forces were removed they returned either wholly or partially to their original shape and volume. Such bodies are called *Elastic*.

Copper and lead and putty, on the other hand, were found to exert no force tending to restore their shape or

DEFINITION. **Elasticity is that property of matter in virtue of which a body requires force to change its shape or volume, and requires a continued application of the force to maintain the shape or volume, and springs back when the force is removed.**

If a body in the form of a sphere be subjected to a uniform pressure over its surface, it will diminish in *volume*, but *its shape* will be *unaltered*; and *if it exerts* force resisting the change, it is said to possess elasticity of volume. Cork, for example, possesses considerable elasticity of volume; it may be compressed to one-eighth its volume, and will recover its original volume.

If a body in the form of a cube be subjected to equal and opposite forces acting along two parallel faces, its shape will be changed, but its volume will be unaltered. The forces producing such change of shape are called *Shearing Forces*, and if the body resists the change it is said to possess elasticity of shape. India rubber possesses considerable elasticity of shape, as it can be stretched to eight times its original length with little change of volume, and will recover its original length.

DEFINITION. **Plasticity is that property of matter in virtue of which a body, when strained by force, does not exert a force tending to recover its shape or volume.**

Plastic bodies are divided into three groups:

(*a*) *Ductile* bodies, such as copper, which can be drawn out in the form of a wire.

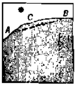
(*b*) *Malleable* bodies, such as lead, which can be beaten out into sheets.

(c) *Soft* bodies, such as putty, which can be moulded into any shape.

It is probable that all bodies are elastic for very small forces and small displacements. Copper is elastic for very small displacements, so also is lead. Again, it is probable that after undergoing certain change of shape or form all bodies become plastic. A steel wire is elastic even when under considerable force, but when the force has reached a certain limit the elastic force of recovery gradually diminishes, and finally the body becomes altogether plastic. There is apparently, in the case of such bodies as glass or crystals, no period of plasticity. Glass and crystals are very elastic, and when they cease to be elastic they break. Such bodies are called **Brittle.**

3. Solids, Liquids, and Gases. It is a matter of every day experience that when water is poured into a vessel it immediately takes up the shape of the vessel. If pitch be placed in a vessel, and be allowed sufficient time, it will also take up the shape of the vessel. But if we place a piece of soft tallow in the vessel its shape will be very slightly altered, whatever length of time be allowed. In fact, in the case of water, there is very little resistance due to the water itself to one part sliding over the other, and what little there may be is never sufficient to prevent the sliding taking place; and ultimately the resistance diminishes to zero, when the sliding ceases. In the case of pitch the resistance to change of shape, i.e. to one part sliding over the other, is greater than in water; but it is never great

enough to prevent a force however small from causing motion, and slowly one part slides over the other, until the forces causing the motion either vanish or are balanced by external forces; and when this takes place the resistance to change of shape also vanishes, and therefore the body does not tend to return to its original shape. *It possesses no elasticity of shape.* To make this clearer, consider a piece of pitch placed in a



vessel. Suppose the part ACB to be divided from the rest by the plane AB. Now, the weight of ACB will cause the portion ACB to slide down the plane AB if the tangential (frictional) resistance that the neighbouring parts exert on ACB is not great enough to prevent the motion. The *resistance to sliding, i.e. the tangential* force along AB, in pitch, water, mercury, and many other substances, is always less than the force (such as the weight of the part or external force) acting on the part; and if the force tending to produce sliding diminishes, so does the tangential force. It follows from this that the free surface of such bodies must be horizontal, or otherwise one part will slide down over the other. It also follows that when such a body is at rest the force between two adjacent parts is perpendicular to the surface of separation. These bodies are called *Fluids,* as distinguished from bodies which exert a resistance to change of shape, and which are called *Solids*.

DEFINITION. **A Solid is a body which exercises a permanent resistance to change of shape, such**

resistance not vanishing as the force producing the change is made smaller and smaller.

DEFINITION. **A Fluid is a body which exercises no permanent resistance to change of shape; it will therefore ultimately take up the shape of any vessel in which it is placed.**

Fluids are divided into two classes—**Liquids and Gases:**

(1) **A Liquid is a fluid the volume of which does not increase indefinitely as the force upon its surface is reduced indefinitely, and therefore a liquid will take the shape of any vessel in which it is placed, and will have a free surface.**

(2) **A Gas is a fluid the volume of which increases continually as the pressure upon it is reduced; it therefore has no free surface, but will fill completely any vessel in which it is placed.**

The **Rigidity** of a body is that property of the body in virtue of which it resists change of *shape*. It is therefore a property peculiar to solids.

If the resistance a solid exerts to change of shape could be so great that no force however large could alter its shape, the solid would be *perfectly rigid*. No solid is perfectly rigid, but many solids require very great force to produce an appreciable change of shape. Such solids may be considered *perfectly rigid* when the forces acting upon them are comparatively small.

All bodies exert a resistance to change of volume, *i.e.* all bodies exert resistance to compression or extension. The degree of **compressibility** or **extensibility** of bodies varies very greatly. As a rule solids and liquids require great force to alter their volume, whereas in the case of a gas the force required is very much less. On this account a gas is said to be a compressible fluid, and a liquid

4. On Viscosity. If the experiments on the extension of india rubber, steel wire, or copper wire be repeated, it will be observed that the body continues stretching for some time after the weight is put on. And when the weight is taken off it takes the body some time to return to its original shape. If, as in these examples, the change or shape of body when acted upon by constant force depends upon the time during which the force acts, the body is said to be *Viscous*. The following experiment will show the great difference between fluids in this respect. Take a funnel and fill it with glycerine up to a given mark, and observe the time it takes to run out. Now fill the same funnel with water, and observe the time it takes the water to run out. It will be at once seen that the resistance to change of shape is greater in glycerine than in water. The same difference is seen by noticing the time it takes a small body to fall in glycerine and in water. Pitch is an extremely viscous fluid, so also is tar. Water and mercury are not very viscous, and sulphuric ether is still less so. Water, mercury, and sulphuric ether offer little resistance to change of shape when one part is sliding over the other; therefore, whether the parts of the fluid are in motion or not, the tangential or frictional resistance is small. Such fluids approach the condition of a perfect fluid which satisfies the following definition:

DEFINITION. **A Perfect Fluid is a fluid in which the resistance to change of shape is always zero.**

When *any* fluid is at rest the tangential resistance to change of shape is zero; and therefore when a fluid

is at rest the force between two adjacent parts is perpendicular to the surface of separation. This is the fundamental property of a fluid, and will be referred to again in Chap. III.

EXAMPLES ON CHAP. I.

1. State exactly what is meant by a "foot," a "cubic :h," a "centimetre."

2. Express the following in metric measure:

3 yds. 10 ft. 6 in. in cm.
10 mi. 800 yds. in kilometres.
6 cubic yds. 10 cubic feet in cc.
30 inches in mm.
32 feet in cm.

3. Express the following in English measure:

760 mm. in inches.
25850 cm. in yards, feet, inches.
3000 cc. in cubic inches.
400 litres in cubic feet.
981 cm. in feet.

4. What is the difference in the behaviour of a steel 1 a copper wire when stretched by a weight?

5. A body is given to you in the form of a wire. How uld you prove if it were elastic or not?

6. Describe the behaviour of a steel wire as it is etched by a slowly increasing force.

7. Prove from the definition that the free surface of a id must be horizontal.

8. State the difference between a solid and a fluid

9. Explain why a soft solid like tallow can be made to lie in a heap, whilst a piece of hard pitch (a liquid) will ultimately spread out so that its surface is horizontal.

10. Does a viscous fluid exert any resistance to change of shape? How does it differ from a solid?

11. A lump of soft tallow can be made to support a heavy piece of iron, whilst a lump of pitch cannot prevent the iron ultimately penetrating it. Explain this.

12. Define a fluid, and deduce from the definition that in a fluid at rest the force on any plane area is perpendicular to the area. Why is this not the case when the fluid is in motion?

13. Distinguish between a liquid and a gas.

14. Glass is elastic, brittle, viscous, and compressible. Explain these terms.

CHAPTER II.

ON SPECIFIC GRAVITY AND DENSITY.

1. The **mass** of a body is the quantity of matter the body contains; it is measured by comparing it with the standard quantity of matter.

The standard quantity of matter adopted in England is the **Pound Avoirdupois.** This standard is quite arbitrary. It is defined to be the quantity of matter which is contained in a bar of platinum deposited in the Offices of the Board of Trade, London, and marked "P.S. 1844, 1 lb." Copies of this, and multiples and sub-multiples of it, are made, and are usually known as "weights."

The metric standard of mass is the **Kilogramme,** and is defined as the quantity of matter contained in a bar of platinum, made by Borda, and deposited in the French Archives. This standard was supposed to represent the mass of 1000 cc. of pure distilled water at a temperature of 4° C., and it does so very nearly, if not exactly.

The British pound is divided into sixteen equal parts, each part being called an ounce. It is also divided into 7000 equal parts, each part being called a grain. Certain well-known multiples of the pound have the names hundredweight, ton, etc.

The multiples or sub-multiples of the metric unit of mass are based on the decimal system. For example, the thousandth part of a kilogramme is called the gramme, and the thousandth part of a gramme is called the milligramme. The convenience of this system as contrasted with the British system is so obvious that the metric system of units is now generally adopted in laboratories.

The British and French units of mass have been accurately compared, and it has been found that the pound Avoirdupois contains 453·593 grammes nearly.

2. The **weight** of a body is the force with which the earth attracts the body. It is proved experimentally that the weight of a given mass is independent of the kind of matter of which the body is composed, but at the same place it is proportional to the mass of the body. By means of an ordinary beam balance the weight of a body can be compared with the weight of a standard mass. If therefore the weight of the unit mass is known, the weight of the body can be found at once. Since the weight of a body is proportional to its mass, the balance also directly compares the mass of the body with the mass of the standard, and therefore the mass of the body is found directly by the balance. Hence the operation of **weighing with a balance** gives two quantities: (1) the weight of the body in terms of the weight at the particular place of the unit mass; (2) the mass of the body in terms of the unit mass. In order to find, however, the weight of the body it would be necessary to perform the difficult dynamical experiment

f finding the weight of the standard mass at the par-icular place. Therefore the quantity directly measured y the balance is the mass of the body, and thus the nass of a body can be obtained by the process of "weighing" with a beam balance.

The unit of weight adopted is the weight of the unit of mass. 'hus the unit of weight may be the weight of the pound Avoirdu-ois, the weight of the kilogramme, or the weight of any multiple r sub-multiple of these. The weight of the pound will be written wt.; the weight of the gramme gmwt.; the weight of an ounce zwt.; and so on. The student of Dynamics learns the importance f distinguishing between the mass of a body and the weight of the ody, but in Hydrostatics the importance of the difference is not so pparent. Such an expression as "a cubic foot of water weighs 000 oz." conveys two facts: (i) the weight of the body; (ii) the iass of the body. The weight of the body is equal to the weight of 000 oz., and is therefore equal to 1000 ozwt.; the mass of the body equal to 1000 oz.

It is sometimes *convenient*, especially in theoretical work, to take he weight of unit volume of a substance as the unit of weight. 'hus the weight of one cubic foot of water is taken as the unit of 'eight in Props. I. II. III. following, *i.e.* 1000 ozwt. is the unit of 'eight. If the weight of one cubic centimetre of water were taken s the unit of weight, the unit of weight would be 1 gmwt.; but iis is a unit of weight which is often taken for other reasons. his is another illustration of the convenience of the French system f units.

3. **Specific Gravity.** If the same volume, say a ubic inch, of different substances be taken, it will be ound that their weights and masses are not the same. he following experiments will make this clear. Take a nall cube of iron, the edge of which is one inch, and imilar cubes of other solids, say lead and pinewood. Make a cubical vessel of brass, the inside measure of which is an inch cube. Find the weight of the cubic inch

of iron, of lead, and of wood. Find also the weight of water the vessel will contain, *i.e.* the weight of one cubic inch of water. It will be found that the iron weighs $4\frac{1}{3}$ oz., the lead $6\frac{1}{2}$ oz., the wood ·289 oz., and the water ·578 oz. Again, take a small cube of iron, the edge of which is exactly one centimetre, and similar cubes of lead and pinewood. Make a cubical vessel of brass, the inside measure of which is a centimetre cube. Find the weight of the iron, the lead, the pinewood, and of the water which fills the little vessel. In each case there will be a cubic centimetre of the substance. It will be found that the cubic centimetre of iron weighs 7·5 gm., of lead 11·3 gm., of wood ·5 gm., and of water 1 gm. Both experiments show that iron is $7\frac{1}{2}$ times as heavy as the same volume of water, lead 11·3 times as heavy, and wood half as heavy. It is convenient to express the weight of a given volume of a substance in terms of the weight of an equal volume of some standard substance. Iron and lead are said to be *specifically* heavier than water, pinewood specifically lighter. *The number which expresses how many times the weight of a given volume of a substance contains the weight of an equal volume of the standard substance is called the "specific gravity" of the substance.*

DEFINITION. **The specific gravity of a substance is the ratio of the weight of a given volume of the substance to the weight of an equal volume of some standard substance.**

In selecting a standard substance attention should be paid to the following conditions: (1) the substance

ought to be universally and easily obtained; (2) it ought to be capable of being obtained in great purity.

Pure distilled water fulfils these conditions, and if we always take it at the same temperature the weight of a cubic foot, or of a cubic centimetre, will always be invariable (at the same place). The standard substance is therefore usually pure distilled water at 4° C., and a cubic foot of water at 4° C. is found to weigh nearly 1000 oz.; a cubic centimetre of water is found to weigh 1 gramme.

Taking water as the standard substance, the specific gravity of iron is $7\frac{1}{2}$, *i.e.* 1 cubic inch of iron weighs $7\frac{1}{2}$ times the weight of 1 cubic inch of water, and 100 cc. of iron weighs $7\frac{1}{2}$ times the weight of 100 cc. of water, or generally any volume of iron weighs $7\frac{1}{2}$ times the weight of an equal volume of water.

In the case of gases it is usual to take hydrogen at 0° C. and 760 mm. of mercury pressure as the standard substance; because, with water as standard, the specific gravity of gases would all be expressed by very small fractions.

The following examples will explain this part of the subject, and the student is recommended to study them carefully:

(i) Sixteen cubic feet of copper is found to weigh 142400 oz. What is the specific gravity of copper?

1 cubic foot of water weighs 1000 oz.
16 cubic feet " " 16000 oz.
$\therefore$ sp. gr. of copper $= \frac{142400}{16000} = 8.9$.

(ii) The specific gravity of gold is 19·3. What is the weight of a piece of gold, the volume of which is 432 cubic inches?

1 cubic foot of water weighs 1000 oz.

1 cubic inch " " $\frac{1000}{1728}$ oz.

432 cubic inches " " $\frac{432 \times 1000 \text{ oz.}}{1728} = 250$ oz.

∴ weight of 432 cubic inches of gold = 19·3 × 250 oz.
= 4825 oz. = 301 lb. 9 oz.

(iii) What volume of iron must be taken to weigh 10 tons? (The specific gravity of iron being $7\frac{1}{2}$.)

We must first find what volume of water will weigh 10 tons.

Now 10 tons = 10 × 20 × 112 × 16 oz. = 358400 oz.

1000 oz. = weight of 1 cubic foot of water.

1 oz. = " $\frac{1}{1000}$ cubic foot of water.

358400 = " $\frac{358400}{1000}$ cubic feet.

= " 358·4 cubic feet.

But iron is $7\frac{1}{2}$ times heavier than water,

∴ Volume of iron = 358·4 ÷ $7\frac{1}{2}$ cubic feet,
= 47·786 cubic feet.

(iv) What is the specific gravity of a substance of which 330 cc. weighs 280·5 gm.?

330 cc. weighs 280·5 gm.

∴ 1 cc. " $\frac{280 \cdot 5}{330}$ gm.

i.e. 1 cc. weighs ·85 gm.

But 1 cc. of water weighs 1 gm.,

∴ 1 cc. of the substance is ·85 times heavier than 1 cc. of water

∴ sp. gr. of the substance = ·85.

(v) What is the weight of 432 cc. of gold?

Sp. gr. of gold is 19·3.

1 cc. of water weighs 1 gm.

∴ 1 cc. of gold weighs 19·3 gm.

∴ 432 cc. " " 8337·6 gm.

(vi) What is the volume of 10 kilogrammes of iron?

10 kilogrammes $= 10 \times 1000 = 10000$ gm.

Since sp. gr. of iron is 7·5,

7·5 gm. is the weight of 1 cc. of iron.

1 gm. „ „ $\frac{1}{7 \cdot 5}$ cc. of iron.

10000 gm. „ „ $\frac{10000}{7 \cdot 5}$ cc. of iron.

Volume of iron $= 1333 \cdot 3$ cc.

EXERCISES.

1. Find specific gravities of following substances which

(a) 12 cubic feet weighs 90000 oz.
(b) 12 cubic inches weighs 3·47 oz.
(c) 5½ cubic feet weighs 3888 lb.
(d) ·81 cc. weighs 834·3 gm.
(e) 5 litre weighs 3650 gm.
(f) 856·6 cc. weighs 17·9865 kilogrammes.

2. Find the weights of

(a) 4 cubic feet of iron.
(b) 5 cubic yards of copper.
(c) 127 cubic inches of alcohol (sp. gr. ·85).
(d) 750 cc. of lead (sp. gr. 11·3).
(e) 3 cubic metres of gold.
(f) 655 litres of glycerine (sp. gr. 1·25).

3. Find the volumes of

(a) 805 lb. of copper.
(b) 301 lb. of gold.
(c) ·5 tons of iron.
(d) 865·5 gm. of copper.
(e) 5 miligrammes of brass (sp. gr. 8·5).
(f) 75 kilogrammes of wood (sp. gr. ·5)

PROP. I.—*To prove the formula $W = VS$.*

For simplicity, take water as the standard substance, and a cubic foot the unit of volume.

Let V cubic feet be the volume of the body.

Let S be the specific gravity of the body.

1 cubic foot of water weighs 1000 oz.

$\therefore$ V cubic feet of water weighs $V \times 1000$ oz.

$$\text{Now } S = \frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{weight of body}}{V \times 1000 \text{ oz.}};$$

$\therefore$ Weight of body $= SV$ 1000 ozwt.

If weight of 1 cubic foot of water be taken as unit of weight, then

weight of body $= SV$ units of weight.

$\therefore$ If $W =$ number of units of weight the body contains

$$W = SV.$$

NOTE 1.—This formula must be carefully understood. The weight of a body is not SV, but SV times the particular unit of weight.

If water is the standard substance,

$$\text{weight of body} = V.S\ 1000 \text{ ozwt.}$$
$$= Vw,$$

where $w =$ weight of 1 cubic foot of the substance.

NOTE 2.—If the C.G.S. system is adopted,

$w =$ weight of 1 cubic centimetre of water,

$=$ weight of 1 gramme.

$\therefore$ Weight of body $= VS$ gmwt.

PROP. II.—*To find the weight and specific gravity of a mixture of any number of substances whose volumes and specific gravities are known.*

Let V_1, V_2, V_3 ... be the volumes of the substances whose specific gravities are respectively S_1, S_2, S_3 ....

The weights of the substances are V_1S_1, V_2S_2, V_3S_3 ... units of weight.

$\therefore$ Weight of mixture $= V_1S_1 + V_2S_2 + V_3S_3 + \ldots$ units of weight, and volume of mixture $= V_1 + V_2 + V_3 + \ldots$

$\therefore$ If S be the specific gravity of the mixture, weight of mixture $= (V_1 + V_2 + V_3 + \ldots) S$ units of weight.

$$\therefore S(V_1 + V_2 + V_3 + \ldots) = V_1S_1 + V_2S_2 + V_3S_3 + \ldots,$$

$$S = \frac{V_1S_1 + V_2S_2 + V_3S_3 + \ldots}{V_1 + V_2 + V_3 + \ldots}.$$

NOTE.—If by chemical combination the volume of the mixture is changed to V, then $SV = V_1S_1 + V_2S_2 + V_3S_3 + \ldots$.

PROP. III.—*To find the volume and specific gravity of a mixture of any number of substances whose weights and specific gravities are known.*

Let W_1, W_2, W_3 ... be the number of units of weight contained by the substances whose specific gravities are respectively S_1, S_2, S_3 ....

The volumes of the substances are $\frac{W_1}{S_1}$, $\frac{W_2}{S_2}$, $\frac{W_3}{S_3}$ ....

$$\therefore \text{Volume of the mixture} = \frac{W_1}{S_1} + \frac{W_2}{S_2} + \frac{W_3}{S_3} + \ldots,$$

and weight of mixture $= W_1 + W_2 + W_3 + \ldots$.

$\therefore$ If S be the specific gravity of the mixture,

$$S\left(\frac{W_1}{S_1} + \frac{W_2}{S_2} + \frac{W_3}{S_3} + \ldots\right) = W_1 + W_2 + W_3 + \ldots.$$

Ex. I.—What is the weight of a cylindrical column o[f] mercury, the height of which is 30 inches, and the area o[f] [i]ts principal section 1 square inch? (Specific gravity o[f] mercury = 13·596.)

If a cylinder be cut by a plane perpendicular to its axis, the [fi]gure formed by the cutting plane is a principal section. If the

area of this section is *one* square inch, then the volume of a cylinder *one* inch long will be *one* cubic inch; if the cylinder be h inches long, its volume will be h cubic inches; and if its base contain a square inches, its volume will be ha cubic inches.

In the example the cylinder is 30 inches long; therefore its volume is 30 cubic inches.

Weight of 30 cubic inches of mercury

$$= \tfrac{30}{1728} \times 13{\cdot}596 \times 1000 \text{ ozwt.} = 236 \text{ ozwt.} = 14\tfrac{3}{4} \text{ lbwt. nearly.}$$

Ex. 2.—Find how many cubic inches of alcohol (specific gravity = ·8) will weigh 15 lb.

We have, weight of body $= VS\,1000$ ozwt.

$$\therefore 15 \times 16 = V \times {\cdot}8 \times 1000.$$

$$\therefore V = \frac{15 \times 16}{{\cdot}8 \times 1000} \text{ cubic feet.}$$

$$= \frac{15 \times 16 \times 1728}{{\cdot}8 \times 1000} \text{ cubic inches.}$$

$$= 518{\cdot}4 \text{ cubic inches.}$$

Ex. 3.—If a volume of 5 cubic feet of a liquid of specific gravity ·8 be mixed with a volume of 7 cubic feet of another liquid of specific gravity ·7, find the specific gravity of the mixture.

Weight of 1st liquid $= 5 \times {\cdot}8 \times 1000$ ozwt. $= 4000$ ozwt.

Weight of 2nd liquid $= 7 \times {\cdot}7 \times 1000$ ozwt. $= 4900$ ozwt.

$\therefore$ Weight of mixture $= 8900$ ozwt.

Volume of mixture $= 7 + 5 = 12$ cubic feet.

$$\therefore \text{Specific gravity of mixture} = \frac{8900}{12 \times 1000} = \tfrac{89}{120} = 74 \text{ nearly.}$$

Ex. 4.—If 5 lb. of a liquid of specific gravity ·8 be mixed with 7 lb. of another liquid of specific gravity ·7, find the specific gravity of the mixture.

$$\text{Volume of 1st liquid} = \frac{5 \times 16}{{\cdot}8 \times 1000} \text{ cubic feet.}$$

$$\text{Volume of 2nd liquid} = \frac{7 \times 16}{{\cdot}7 \times 1000} \text{ cubic feet.}$$

$$\text{Volume of the mixture} = \frac{16}{1000}\left(\frac{5}{\cdot 8} + \frac{7}{\cdot 7}\right) \text{ cubic feet.}$$

$$= \tfrac{16}{1000} \times (6\cdot 25 + 10) \text{ cubic feet;}$$

$$= \tfrac{16}{1000} \times 16\cdot 25 \text{ cubic feet.}$$

$$\text{Weight of mixture} = 5 + 7 \text{ lbwt.} = 12 \text{ lbwt.}$$

$$\therefore \text{Specific gravity of mixture} = \frac{12 \times 16}{16 \times 16\cdot 25} = \cdot 738.$$

Ex. 5.—A body displaces 250 gm. of a liquid, the specific gravity of which is ·85. What weight of water would it displace?

·85 gm. is the weight of 1 cc. of the liquid.

$\therefore$ 1 gm. " " $\frac{1}{\cdot 85}$ cc. "

250 gm. " " $\frac{250}{\cdot 85}$ cc. "

i.e. 294·11 cc. nearly,

$\therefore$ Volume of the body is 294·11 cc.

And it would displace 294·11 gm. of water.

Ex. 6.—An alloy of copper (sp. gr. 8·8) and lead (sp. gr. 11·3) is found to weigh 314 gm.; the volume is 30 cc. Find the volume of each component.

Let x cc. = volume of the copper;

y cc. = volume of lead.

$\therefore$ Weight of the copper $= x \times 8\cdot 8$ gmwt.

Weight of the lead $= y \times 11\cdot 3$ gmwt.

Hence $x \times 8\cdot 8 + y \times 11\cdot 3 = 314$,

and $x + y = 30$.

Solving these simultaneous equations

$x = 10$,

$y = 20$.

$\therefore$ *Volume of copper is 10 cc. and volume of lead 20 cc.*

Weight of copper is 88 gmwt. and weight of the lead is 226 gmwt.

$\therefore$ The percentage by volume is $\frac{10}{30} \times 100$, *i.e.* 33·3̇ per cent. for copper.

The percentage by volume is $\frac{20}{30} \times 100$, *i.e.* 66·6̇ per cent. for lead.

The percentage by weight is $\frac{88}{314} \times 100$, *i.e.* 28 per cent. nearly for copper.

The percentage by weight is $\frac{226}{314} \times 100$, *i.e.* 72 per cent. nearly for lead.

EXERCISES.

1. Find the specific gravity of a mixture of 3 cubic feet of a liquid of sp. gr. 1·25 with 5 cubic feet of a liquid of sp. gr. ·85.

2. Find the sp. gr. of a mixture of 3 cc. of a liquid of sp. gr. 1·25 with 5 cc. of a liquid of sp. gr. ·65.

3. If 25 gm. of a liquid of sp. gr. ·8 he mixed with 38 gm. of a liquid of sp. gr. 1·2, find the sp. gr. of the mixture.

4. What is the weight of a copper wire 100 cm. long and 1 sq. mm. sectional area?

5. What is the length of a column of mercury of 1 sq. inch sectional area which weighs 28 lb.?

4. By the process of "weighing" it is easily found that the mass of one cubic inch of a body varies with the kind of the matter. For example, the mass of 1 cubic inch of iron is $4\frac{1}{3}$ oz.; of 1 cubic inch of lead, $6\frac{1}{2}$ oz.; of 1 cubic inch of water, ·578 oz. This is expressed by saying that bodies have different *densities*.

DEFINITION. **The density of a substance is the mass of unit volume of the substance.**

Hence density is measured in units of mass per unit volume. If the unit mass is 1 lb. and the unit volume 1 cubic foot, the unit density is 1 lb. per cubic foot. Since 1000 oz. = 62·5 lb., the density of water is nearly 62·5 lb. per cubic foot; of iron, 468·75 lb. per cubic foot. If the unit of mass is 1 gm. and the unit of volume 1 cc., the unit of density is 1 gm. per cc.; therefore the density

of pure distilled water at 4° C. is 1 gm. per cc.; of iron, 7·5 gm. per cc.

If the density of a body is D lb. per cubic foot, and the volume is V cubic feet, then the mass of the body is $V.D$ lb. If the mass is M lb, then $M = VD$.

Since the specific gravity of a substance is the ratio of the weight of (say) 1 cubic foot of the substance to the weight of 1 cubic foot of water, it is also equal to the ratio of the mass of 1 cubic foot of the substance to the mass of 1 cubic foot of water, *i.e.* it is equal to the ratio of the density of the substance to the density of water. This density ratio is sometimes given as the definition of specific gravity; it is also sometimes called relative density, and sometimes, unfortunately, abbreviated to "density."

Specific gravity may be defined as *the ratio of the mass of a given volume of the substance to the mass of an equal volume of some standard substance*, and Propositions, corresponding to those given above on the relation between the weight and volume of bodies, can be given for the identical relation between the mass and volume of the same bodies. All that is necessary is to substitute in § 3 the word "mass" for "weight" wherever the latter may occur. The same remark applies to experiments on pp. 27—32, and to the examples at the end of the chapter.

If Δ lb. per cubic foot is the density of water and S the specific gravity (= density ratio) of a substance, then $S\Delta$ = density of the substance. If the volume is V cubic feet, then the mass of the body is $V.S.\Delta$ lb. If this is M lb., then $M = V.S.\Delta$.

The mass of 1 cub. ft. of the substance $= S\Delta$ lb.

$\therefore$ weight of 1 cub. ft. of the substance $= S\Delta$ lbwt.

$\therefore$ weight of V cub. ft. of the substance $= V.S.\Delta$ lbwt.

$S\Delta$ is what is denoted by w in Note 1, page 19.

The volume of unit mass of a substance is called the **specific volume** of the substance. For example, the volume of 1 lb. of water at 4° C. is ·016 cubic feet; therefore the specific volume is ·016 cubic feet per lb. The volume of 1 lb. of saturated steam at 100° C. is 26·36 cubic feet, so that its specific volume at 100° C. is 26·36 cubic feet per lb. The volume of 1 gramme of hydrogen at normal temperature and pressure is 11·16 litre: therefore specific volume of the hydrogen is 11160 cc. per gm.

NOTE.—Throughout this book, except where expressly stated to the contrary, the substance is supposed homogeneous, so that every equal volume of it will have the same mass.

EXPERIMENTS ON CHAPTER II.

TO FIND THE DENSITY, VOLUME, AND SPECIFIC GRAVITY OF BODIES BY DIRECT METHODS.

In order to find experimentally the **density** of a substance, two quantities must be measured:

1. The mass of the body.
2. The volume of the body.

1. The mass of the body can be found by the process of "weighing."

2. The volume of the body can be found

(1) By direct measurements, if the body is of regular form.

(2) By method of displacement of a liquid. (See Exp. III. of this Chapter.)

(3) By Hydrostatic methods. (See Chapter IV.)

If the mass of the body is found to be M gm., and

the volume V cc., then the mass of one cubic centimetre is $\frac{M}{V}$ gm., therefore the density is $\frac{M}{V}$ gm. per cc.

EXP. I.—*To find the mass of one cubic centimetre of iron,* i.e. *to find the density of iron in grammes per cubic centimetre.*

i. To find the **volume** of the iron solid it is necessary to take a solid of regular shape. Suppose the body taken is cylindrical. To find the volume of a cylinder, two lengths must be measured, viz. the length and the diameter.

(1) *To find the length.* Place the metre scale with its graduated edge along the length of the cylinder, so that it touches the cylinder throughout its whole length. Read the length of the cylinder. With care the length can be estimated to at least quarters of a millimetre. Make three such measurements, and take the mean.

(2) *To find the diameter.*

1st Method. Find the centre of one end, and measure the length of three different diameters, and find the mean.

2nd Method. Find the *circumference,* and since the circumference is 3·1416 times the diameter, the diameter is found by dividing the circumference by 3·1416. To find the circumference, take a sheet of paper and draw a straight line on it. Now mark a point on the edge of one end of the cylinder, and starting with this point at one end of the line, roll the cylinder (taking care that it does not slide) with the edge of one end along the line, and until the cylinder has made (say) three revolutions, then mark on the line the new position of the marked point of the cylinder. Measure the length from the end up to this point. This length will be three times the circumference. Hence the circumference and therefore the diameter can be found.

ii. To find the **mass** of the cylinder. This can be done by "weighing" the cylinder.

Example.

(1) Length of cylinder 3·82, 3·81, 3·81 cm.; mean, 3·813 cm.

(2) Diameter: 1st Method. 3·00, 3·00, 3·00 cm.; mean, 3·00 cm. 2nd Method. Three times the circumference 28·31, 28·35, 28·32 cm., *i.e.* mean diameter 3·005 cm.

(3) Mass of cylinder 206·5 gm.

Hence the mean diameter is 3 cm.

$\therefore$ Volume of cylinder $= 3{\cdot}1416 \times (1{\cdot}5)^2 \times 3{\cdot}813$ cc.
$= 26{\cdot}95$ cc.

$\therefore$ 26·95 cc. of iron weighs 206·5 gm.

$\therefore$ 1 cc. " " $\frac{206{\cdot}5}{26{\cdot}95}$ gm.

i.e. 1 cc. of iron weighs 7·66 gm.

$\therefore$ the density of iron is 7·66 gm. per cc.

NOTE 1.—In the same way the weight of one cubic inch of iron can be found, using a scale of inches and decimals of an inch.

NOTE 2.—A convenient solid to take is a sphere, where only one measurement is required—the diameter. The diameter of a sphere can be found by a small pair of calipers; or, if the sphere is very small by a wire gauge.

EXP. II.—*To find the weight of one cubic centimetre of water or of any liquid.* 1st Method.

The principle of this method is to take a vessel of such a shape that its volume up to a given mark can be found by direct measurements (a cylindrical vessel is convenient); and then to find the weight of liquid which will fill the vessel up to the mark. In this way the weight (and mass) of a known number of cubic centimetres of the liquid is found, and therefore the weight (and mass) of one cubic centimetre of the liquid can be calculated. If, for example, the volume of the vessel found by direct measurements is 100 cc., and the weight of the liquid is 125 gmwt., then the weight of 1 cc. of the liquid is 1·25 gmwt.

i. Take a hollow cylindrical vessel with a flat bottom, and find its depth, and internal diameter as in Exp. I. Hence find its internal volume.

ii. Find the weight of water it will contain.

(1) Weigh the vessel empty.

(2) Carefully fill it with water up to the brim, and find the weight of vessel full of water.

The difference between (1) and (2) is the weight of the water which the vessel contains.

Example.

i. (1) Depth of cylinder 3·8, 3·8, 3·81 cm.; mean 3·803 cm.
(2) Diameter of inside 3·0, 3·0, 3·1 cm.; mean 3·033 cm.

ii. (1) Weight of cylinder empty 37·25 gm.
(2) Weight of cylinder full of water 64·6 gm.

Calculation.

Volume of cylinder $= 3·1416 \times (1·516)^2 \times 3·803$ cc.
$= 27·457$ cc.

Weight of water in vessel $= 64·6 - 37·25$, *i.e.* 27·35 gm.

Hence 27·457 cc. of water weighs 27·35 gm.

$\therefore$ 1 cc. " " $\frac{27·35}{27·457}$ gm.

1 cc. " " ·9960 gm.

Exercise 1.—The weight of same vessel filled with glycerine was found to be 71·45 gm. Find the density of glycerine.

Note 1.—To fill the vessel to the brim it is best to almost fill it, and then place it in the scale-pan, and add drops of water by means of a glass tube.

Note 2.—It will be seen that the water will not have quite a flat surface. The vessel ought therefore to be filled until the level of the surface of the water is on the level of the edge.

Exp. III.—*To find the weight of one cubic centimetre of a liquid.* 2nd Method. By displacement.

The principle of this method is to find the weight of liquid displaced by a solid, the volume of which can be calculated from direct measurements. Suppose the

volume of the solid is found to be 10 cc., then it will displace 10 cc. of a liquid when totally immersed in it. Now, if the weight of a liquid displaced by it is found to be 6·5 gmwt., then this is the weight of 10 cc. of the liquid, and therefore the weight of 1 cc. of the liquid is ·65 gmwt.

Take a solid, like the iron cylinder in Exp. I., the volume of which can be found by measurement.

i. Find its volume.

ii. To find the weight of water it will displace.

Take a small vessel in which the solid can be placed; a narrow tin vessel will do. Make a fine mark inside the vessel near the top.

(1) Fill the vessel with water up to the mark, and *place it with the solid in the scale-pan*, and find their weight.

(2) Now pour out the water, and place the solid inside the vessel, and pour in water up to the same mark, and find the weight of the whole.

The difference between (1) and (2) will be the weight of water displaced by the solid, and since its volume has been found, the weight of 1 cc. of water can be calculated.

Example.

Weight of vessel, water, and iron outside 388·0 gmwt.

Weight of vessel, water, and iron inside 361·15 gmwt.

∴ weight of water displaced by the solid = 388·0 − 361·15 gmwt.

i.e. 26·85 gmwt.

Volume of solid (as in Exp. I.) found to be 26·95 cc. Hence volume of water displaced = 26·95 cc.

∴ 26·95 cc. of water weighs 26·85 gm.

$$\therefore 1 \text{ cc. of water weighs } \frac{26{\cdot}85}{26{\cdot}95} \text{ gm.}$$

i.e. ·9963 gm.

NOTE 1.—The mark inside the vessel should be made with a steel point, and should be horizontal when the vessel is standing on its base. The water can be added by means of a piece of glass tubing.

NOTE 2.—*When the solid is placed inside, care must be taken that no globules of air adhere to its surface.* If any bubbles are seen, they can be removed by a piece of tubing.

EXP. IV.—*To find the volume of any solid by displacement.*

The principle of the method of finding the volume of any solid by displacement is this:

(1) The weight of one cubic centimetre of a liquid is known or found, and therefore the volume of 1 gmwt. is calculated.

(2) The weight of the liquid displaced by the solid is found, and from (1) the volume of this liquid is calculated. But the volume of this liquid is equal to the volume of the solid.

(*a*) *If the solid does not dissolve in water.*

Find the weight of water displaced by the solid as in III. ii.

Then, assuming that the volume of 1 gm. of water is 1 cc., the volume of the water displaced, and hence the volume of the solid, can be found.

Example. Solid: copper wire.

(1) Weight of vessel, water, and wire outside 241·9 gmwt.

(2) Weight of vessel, water, and wire inside 234·8 gmwt.

Calculation.

Weight of water displaced by the solid = 241·9 − 234·8 gmwt.
= 7·1 gmwt.

Now 1 gm. is weight of 1 cc. of water.

∴ 7·1 gm. „ 7·1 cc. of water.

∴ Volume of water displaced, and therefore the volume of the solid, is 7·1 cc.

(*b*) *If the solid dissolves in water.*

Instead of water use some liquid which will not dissolve the solid.

Find the weight of 1 cc. of the liquid (Exp. I.).

Find the weight of liquid displaced by the solid (III. ii.) and hence deduce the volume of the solid.

Example.

Weight of vessel, turpentine, and solid outside 195·3 gmwt.
Weight of „ „ and solid inside 191·15 gmwt.
Weight of 1 cc. of turpentine = ·862 gmwt.

Calculation.

Weight of turpentine displaced by solid = 195·3 — 191·15 gmwt.
= 4·15 gmwt.

Now ·862 gm. is weight of 1 cc. of turpentine.

$\therefore$ 1 gm. „ $\frac{1}{\cdot 862}$ cc. of turpentine.

$\therefore$ 4·15 gm. is weight of $\frac{4\cdot 15}{\cdot 862}$ cc. of turpentine, *i.e.* 4·814 cc.

$\therefore$ Volume of solid = 4·814 cc.

(c) *If the solid floats in water,* e.g. *wood.*

Take a solid which tied to the wood will sink it in water. This solid is called a sinker.

Find the weight of water displaced by the sinker (IV. *a*).

Tie the wood and sinker together by a piece of fine cotton. Find the weight of water displaced by the wood and sinker (IV. *a*).

Hence deduce weight of water displaced by the wood alone, and hence the volume of the wood.

Example. Solid: Mahogany.

(1) The sinker used was the copper wire of IV. *a*.
(2) Weight of vessel, water, and both solids outside 253·9 gmwt.
(3) Weight of vessel, water, and both solids inside 228·9 gmwt.

Calculation.

Volume of sinker 7·1 cc. (IV. *a*).

Weight of water displaced by both solids = 25 gmwt.
Volume of sinker and wood = 25 cc.
$\therefore$ Volume of wood = 25 — 7·1 cc. = 17·9 cc.

If the wood is rectangular, measure its three edges, and find its volume directly.

EXP. V.—*To find the specific gravity of a solid.*

To find the **specific gravity** of a solid it is necessary to find

(1) weight of the solid;

(2) weight of an equal volume of water.

The weight of an equal volume of water can be found either (i) by direct measurements if the solid is a regular one; (ii) by displacement methods (Exp. IV.).

If the weight of the solid is W gmwt., and the weight of an equal volume of water is w gmwt., then the specific gravity $= \frac{W}{w}$.

Example. See Exp. IV. *a.*

(1) Weight of solid = 62·3 gmwt.
(2) Weight of water displaced by the solid = 7·1 gmwt.

Hence the specific gravity $= \frac{62 \cdot 3}{7 \cdot 1} = 8 \cdot 77$

Find the specific gravity of the following solids by displacement method, and where they are of regular figure test your result by direct measurement: Iron cylinder, brass cylinder, piece of copper wire, sulphur, paraffin, pine, mahogany, cork, glass marble, an ivory billiard ball.

EXERCISES.

1. Weight of sulphur 27·4, 27·4, 27·45 gm.; weight of vessel and water 187·8, 187·8, 187·55 gm.; weight of vessel, water, and solid inside 201·5, 202·5, 201·7 gm. Find the specific gravity of the sulphur.

2. Weight of cork 3·55, 3·54, 3·55 gm.; weight of vessel, water, cork, and sinker outside 244·35, 244·3, 244·20 gm.; weight of vessel, water, cork, and sinker inside 212·98, 212·90, 213·20 gm.; weight of vessel, water, and sinker outside 241·00, 240·9, 239·6 gm.; weight of vessel, water, and sinker inside 234·0, 233·8, 234·3 gm. Find the specific gravity of the cork.

3. Diameter of ivory ball 5·0, 4·98, 4·99 cm.; weight of the ball 114·2 gm. Find the specific gravity of ivory.

EXP. VI.—*To find the specific gravity of a liquid.*

To find the specific gravity of a liquid it is necessary to find

(1) the weight of a certain volume of the liquid;

(2) the weight of an equal volume of water.

The weights of equal volumes of water and of another liquid can be found either by finding the weights of each which will fill a vessel up to a given mark, or by finding the weights of each displaced by the same solid.

1st Method. Take the narrow tin vessel used in Exp. III., or better, take a small flask with a narrow neck.

(1) Weigh the vessel empty.

(2) Weigh the vessel filled with water up to a given mark.

(3) Weigh the vessel filled with the liquid up to the same mark.

Hence the weights of equal volumes can be found, and the specific gravity can be calculated.

Example. Liquid: Turpentine.

Weight of vessel empty 57·55 gmwt.

Weight of vessel filled with water up to the mark 177·6 gmwt.

Weight of vessel filled with the liquid up to the same mark 160·3 gmwt.

Calculation.

Weight of the water $= 177{\cdot}6 - 57{\cdot}55$ gmwt. $= 120{\cdot}05$ gmwt.

Weight of the liquid $= 160{\cdot}3 - 57{\cdot}55$ gmwt. $= 102{\cdot}75$ gmwt.

And since there is the same volume in each case,

$$\text{sp. gr. of turpentine} = \frac{102{\cdot}75}{120{\cdot}05} = {\cdot}856.$$

2nd Method. Take any solid which will sink both in the liquid and in the water.

(1) Find the weight of water displaced by the solid

(2) Find the weight of the liquid displaced by the same solid (Exp. III.).

Hence will be found the weights of equal volumes of the liquid and of water.

Example.

(1) Weight of vessel, water, and steel cylinder outside 394·5 gm.
(2) Weight of vessel, water, and steel cylinder inside 367·8 gm.
(3) Weight of vessel, turpentine, and steel cylinder outside 366·7 gm.
(4) Weight of vessel, turpentine, and steel cylinder inside 343·5 gm.

Calculation.

From (1) and (2) weight of water displaced = 394·5 — 367·8 gm.
= 26·7 gm.

From (3) and (4) weight of turpentine displaced = 366·7 — 343·5 gm.
= 23·2 gm.

Hence sp. gr. of turpentine $= \frac{23 \cdot 2}{26 \cdot 7} = \cdot 868.$

EXERCISES.

1. Find the density of glycerine by three direct methods.

2. Find the volume of each centimetre length of a glass tube, assuming the specific gravity of mercury to be 13·596.

3. Find the sectional area of a piece of copper wire.

EXAMPLES ON CHAP. II.

1. Find the volume of the following: (1) 1000 gm. of iron; (2) 1256 kilogm. of gold; (3) 505 gm. of mercury; (4) 1567 lb. of silver; (5) 3 tons of wood; (6) 5 cwt. 6 lb. of ice; (7) 5 oz. of air at specific gravity ·0013.

2. Find the weight of the following: (1) 1000 cc. of iron; (2) 1567 cc. of gold; (3) 505·67 cc. of mercury, 1567 cubic feet of iron; (4) 306 cubic metres of air at specific gravity ·0013; (5) 76 cubic feet 144 cubic inches of wood

3. Find the specific gravity of the following substances of which (1) 100 cc. weighs 85 gm.; (2) 155 cc. weighs 193·75 gm.; (3) 27 cc. weighs 237·6 gm.; (4) 5 cubic feet weighs 2343·875 lb.; (5) 27 cubic yards weighs 34000 lb.; (6) 3 cubic feet weighs 3·87 oz.

4. Find the weight of an iron circular cylinder 10 feet long and 2 inches thick.

5. Find the weight of a copper spherical ball 10″ diameter.

6. Find the weight of the rim of a fly-wheel of rectangular section: outer radius 5′ 6″, inner radius 5′, width 8″.

7. A cylinder is made of copper; its length is 10 cm. and the weight is 800 gm. What is the area of its end, and what is the thickness of the cylinder?

8. A tank is $6' \times 4' \times 2\frac{1}{2}'$. How many gallons of water will it contain?

9. A hollow conical vessel 10 ft. deep, with a diameter at the top of 6 ft., is filled with glycerine (sp. gr. 1·25). Find the weight of the glycerine.

10. A wire 1 metre long weighs 10 gm. What is its thickness? Specific gravity of the metal is 8·8.

11. A pellet of mercury is forced into a capillary tube, and is found to occupy 1·1 cm. of it. If the mercury weighs ·0357 gm., what is the diameter of the bore?

12. A spherical ball 10 cm. diameter is found to weigh 1250 gm. Find the specific gravity of the substance.

13. What weight of zinc will be required for 3 dozen zinc cylinders each 9 inches long by $1\frac{1}{2}$ inches thick?

14. What weight of copper will be required for 3 dozen cylindrical copper vessels 9 inches deep and $4\frac{1}{2}$ inches wide (outside measure) thickness of plate $\frac{1}{16}$ inch?

15. A vessel will hold 10 lb. of water. What weight of

16. A bottle weighs 40 gm.; filled with water it weighs 145 gm.; filled with a liquid it weighs 135 gm. Find the specific gravity of the liquid.

17. A bottle weighs 1 lb. 6 oz.; filled with water it weighs 3 lb. 4 oz.; filled with a liquid it weighs 2 lb. 14 oz. *Find the specific gravity of the liquid.*

18. Find the weight of a mixture of 10 cubic inches of gold and 3 cubic inches of silver. What weight of water would the mixture displace?

19. *A piece of gold weighs 105 gm.* What weight of water would it displace? What weight of mercury would it displace?

20. What volume of silver would displace the same weight of water that 6 lb. of gold will displace?

21. What volume of mercury will weigh as much as 8 lb. of iron?

22. What volume of water would weigh as much as 300 gm. of cork?

23. A piece of lead weighs 4 lb. What weight of water would it displace? What volume of cork must be added to it to make the weight of the two together equal to the weight of water they would displace?

24. A piece of copper weighing 7 lb. is placed in one scale-pan of a balance. What volume of platinum must be placed in the other in order to balance it?

25. If the specific gravity of air is taken as ·00129, what will be the weight of a million litres of air? And if the specific gravity of coal gas is ·00052, what volume of coal gas would have the same weight?

26. Find the weight of 1 cubic inch of mercury, of water, of glycerine, and of alcohol.

27. A spherical ball 10 cm. in diameter is sunk respectively in mercury, in glycerine, and in alcohol. What weight of each will it displace?

28. A copper ball 6 inches in diameter is half immersed in water. What weight of water will it displace?

29. What weight of mercury must be poured into a vessel full of water in order to displace 100 gm. of water?

30. Find the mass of 53 cubic inches of steam at each of the following densities expressed in lb. per cubic foot: (1) ·03797; (2) ·1620; (3) ·3664.

31. Find the mass of steam which will fill a cylinder 1 foot long and $8\frac{1}{2}$ inches diameter with steam of density ·1620 lb. per cubic foot.

32. If the density of air is ·00129 gm. per cc., what mass of air will fill a room 40 ft. by 30 ft. by 15 ft.?

33. If the mass of a litre of a gas is ·0896 gm., express its density in lb. per cubic foot.

34. Find the volume of a gm. of a gas, the density of which is ·0000895 gm. per cc.

35. The density ratio of oxygen (hydrogen being the standard) is 16. Find the volume of 100 gm. of oxygen.

36. How many cubic inches of iron weigh a ton?

37. Find the specific gravity of a substance of which 3 cubic feet weigh 5000 oz.

38. Find the weight of 2 cubic inches of wax whose specific gravity is ·96.

39. Find the specific gravity of a substance of which 15 cubic feet weigh 7969 lb.

40. How many cubic feet of copper must be taken to make a weight of 200 lb., and how many to make a weight of 1000 grs.? (Specific gravity of copper = 8·9.)

41. Find the specific gravity of a substance, 16 cubic yards of which weigh $84\frac{3}{4}$ tons.

42. Find the weight of lead required to cover a roof 14 yards long by 12 yards broad, the lead being 2 inches [illegible]

43. What must be the length of a cylinder of water, the area of the principal section of which is 1 square inch, in order that it may weigh $14\frac{3}{4}$ lb.?

44. Two fluids of specific gravities 3 and 5 are mixed together in equal volumes. Find the specific gravity of the mixture. If they are mixed in equal weights, what is the specific gravity?

45. The specific gravities of pure gold and copper being 19·3 and 8·9, find the specific gravity of standard gold, which is an alloy of gold and copper in the proportion of 11 : 1.

46. A pint of spirits of specific gravity 1·84 is mixed with one pint of water. If the mixture measures 2 pints, what is its specific gravity?

47. A bottle filled with water weighs 212 grams; 50 grams of metal filings are thrown in; the overflow of water is removed, leaving the bottle still just filled. The bottle now weighs 254 grams. Find the specific gravity of the metal.

The following examples are important, because they will enable the experimenter to discover the relative importance of the different measurements he makes:

48. *On Exp. I.* Find the density of iron from the following results: (*a*) length of cylinder 3·82 cm., diameter 3·0 cm.; (*b*) length of cylindor 3·813 cm., diameter 3·01 cm.; weight of cylinder in each case 206·5 gmwt.

49. *On Exp. I.* Find the density of iron from the measurements of Example 48, if the weight is taken as 206 gmwt.

50. *On Exp. II.* Find the density of water from the following results: depth of hollow cylinder 3·8 cm., diameter 3 cm., and weight of water 27·35 gmwt.

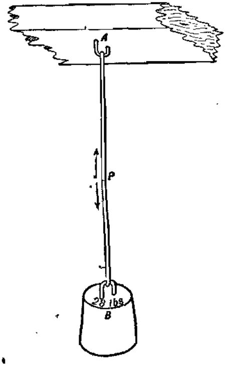
51. *On Exp. II.* If in Exp. II., Example 50, the water was only poured in to within ·1 mm. of the brim, what would the density be calculated to be?

52. *On Exp. III.* If the area of the cross section of the vessel used in this experiment is 12 sq. cm., and the water is poured in to within ·2 mm. of the mark when the solid is inside, what will the weight of water displaced by the solid appear to be, and what will the weight of 1 cc. of water calculated from this be?

CHAPTER III.

ON PRESSURE AT ANY POINT OF A FLUID.

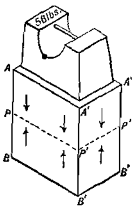
1. Suspend a body by a string, or by a piece of india rubber, as in the figure, where the string is attached to some fixed support at A. Let P be any part of the



string. The part AP of the string is acted upon by a force across a horizontal section at P *downwards* due to the presence of BP, and the part BP is acted upon

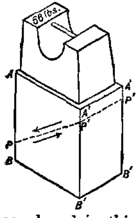
by a force across the section at P *upwards* due to the presence of the part AP. These two forces are equal, and in this case they tend to separate the parts AP, BP further from each other. They constitute what may be called a **Pull.**

Consider a body AB' with a weight placed upon it.



Suppose the body divided by the horizontal plane PP'. Then the lower part PB' is acted upon by a force across PP' *downwards* due to the presence of the upper part AP', and the part AP' is acted upon by a force across PP' *upwards* due to the presence of PB'. These two forces are equal, and in this case tend to bring the parts of the body nearer together. They constitute what may be called a **Thrust.**

If the body is imagined to be divided by a plane



inclined to the vertical, then the upper part AP' will tend to slide over the lower part BP'. There will therefore be a force acting on AP' due to the presence of BP', tending to prevent AP' sliding over BP'. And the part BP' is acted upon by a force due to the presence of AP', tending to make the parts of BP' slide over PP'. These two forces are equal, and in this case, since the forces act along PP', they are called tangential forces, or **Shearing Forces.**

The forces acting across any area, whether thrust, pull, or tangential, are measured in terms of the unit of force. The unit of force which will be generally adopted is either the weight of a pound Avoirdupois, or the weight of one gramme, and in some cases the weight of a column of liquid.

A fluid at rest cannot exert a pull or a tangential force, and hence in hydrostatics the forces throughout the body considered will he thrusts only.

If the thrust across every square inch of an area is the same, it is easy to find the thrust across the area if we know the thrust across one square inch.

The thrust across a unit area is called **Pressure.**

If the thrust is the same on every equal area of a surface, the *pressure on the surface is uniform.*

If the thrusts are not equal on equal areas, the pressure is *variable.*

In a similar manner the pull per unit area is called **Tension.** In elementary Mechanics and Hydrostatics the tension in this limited sense is rarely wanted, so that the word tension is often used for Pull.

We have seen that the action across a given area consists of two equal and opposite forces; when the area is the unit area the system of two equal forces is called a Stress. A **Stress** is therefore two equal and opposite forces acting across a unit area separating two parts of a body, and tending to produce relative displacement of those parts. The displacement produced is called a **Strain.** When the stress is a pressure, the strain produced is *compression.* When the stress is a tension, the strain produced is *elongation.* When the stress consists of two tangential forces, it is called a shearing stress, and the strain produced is called a *shear.*

When a fluid is at rest, the force across any plane is perpendicular to the plane; therefore there is no shearing stress in a fluid. Except in a few special circumstances the stress is a pressure.

2. *When the pressure is uniform.*

If the area of a plane is 12 square yards, and the pressure on it is uniform, and is equal to 15 lbwt. on every square inch, then the thrust on a square foot is 15×144 lbwt. $= 2160$ lbwt.; the thrust on a square yard is 9×2160 lbwt. $= 19440$ lbwt.; and the thrust on the whole area is 12×19440 lbwt. $= 233280$ lbwt.

If 1 foot is the unit of length, and the area of the plane is a, this would mean that the plane contains a square feet. If the weight of 1 lb. is the unit of force, and the pressure is denoted by p, this would mean a pressure of p lbwt. on every square foot, or p lbwt. per square foot; and the force on the whole plane is pa lbwt.

The pressure is therefore always expressed in units of force per unit of area, *e.g.* in lbwt. per square foot, or lbwt. per square inch; or gmwt. per square cm., or dynes per square cm.; and where the unit is not stated the units of area and force are supposed to be known from the context, or to have been fixed from the beginning.

EXERCISES.

1. Find the pressure in lbwt. per square inch when

(*a*) Force on a plane of 3 sq. ft. is 432 lbwt.

(*b*) Force on a plane of 1000 sq. inches is 68 lbwt.

(*c*) Force on an area of 5 sq. yards is 60 tons wt.

(*d*) Force on an area of 10 sq. inches is equal to the weight of 4080 cubic inches of water.

2. Find the pressure in gmwt. per sq. cm. when

(*a*) Force on a plane of 300 sq. cm. is 56 kilogmwt.

(*b*) Force on a circle of 10 cm. diameter is 980 gmwt.

(*c*) Force on an oblong 15 cm. by 10 cm. is 5500 gmwt.

(*d*) Force on an area of 10 sq. cm. is equal to the weight of 76 cc. of mercury.

3. Find the force on each of the following planes:

(*a*) 555 sq. inches under pressure of 15 lbwt. per sq. inch.

(*b*) 1 sq. mile under pressure of 80 lbwt. per sq. foot.

(*c*) 100 sq. cm. under pressure of 560 gmwt. per sq. cm.

3. *When the pressure is variable.*

If the area of a plane is 12 square yards, and it be divided up into 15552 square inches (like a chess board), and the force on each square inch found, then, if the force is not the same on each square inch, the pressure is variable. It will be necessary therefore to find the force on each square inch, and add all the forces together in order to get the thrust on the whole area. If, now, one of these squares is subdivided into nine equal parts, and the force on each part is found to be the same as near as can be determined by measurement, then the pressure on the square inch is nearly uniform, and the force on it can be taken as the pressure *at* any point in the square inch. A similar method is adopted in finding the velocity of a train *at* any moment by finding the space passed over (say by counting the telegraph-posts passed) in one minute; this will give the velocity in feet per minute at the moment, provided that the velocity has not sensibly changed in the minute. If the velocity is changing rapidly (as at start), then, in order to find the velocity at any moment, the space

passed over in a very small time (say a second) must be found, and hence the velocity of the train found in feet per second. This velocity can of course be expressed in miles per hour, or any other suitable unit of velocity In the same way, if the force on a small area surrounding a point is found, the pressure *at* the point can be expressed in lbwt. per square inch, or in any other suitable units of pressure. For example, suppose a small area of $\frac{1}{100}$ square inch surrounding a point is such that if subdivided the force on each equal area of it is practically the same, then the pressure throughout the area is practically uniform. Suppose the force on it is 5 lbwt., then the pressure *at* the point is 5 lbwt. per $\frac{1}{100}$ square inch, *i.e.* at the rate of 500 lbwt. per square inch. Hence the pressure at the point is said to be 500 lbwt. per square inch. This, of course, does not mean that the force on a square inch containing the point is 500 lbwt. any more than the velocity of a train at any moment being 60 miles per hour means that the train will pass over 60 miles in the next hour.

DEFINITION. **The pressure at a point of a fluid is the force on unit area, provided that the force on every indefinitely small area of it is the same as that on an equal small area containing the point.**

In this definition it is tacitly assumed that the pressure at the point is independent of the position of the plane area containing the point, *i.e.* that the pressure *at* the point is the same whatever be the position of the plane, *i.e.* whatever be the direction of the pressure. That this is so depends upon the fundamental

property of a fluid at rest, and can be proved to be a consequence of this property.

THE FUNDAMENTAL PROPERTY OF A FLUID AT REST.—**The pressure at any point of a fluid at rest is always perpendicular to any surface with which it is in contact.**

FUNDAMENTAL LAW OF HYDROSTATICS.—**The pressure at any point of a fluid at rest is the same in all directions about the point.***

This is the characteristic property of a fluid as distinguished from a solid. For instance, if a very hard solid fills a cylindrical vessel, and a pressure be applied to the solid in the direction of the axis of the cylinder, this pressure is only felt on the base and not on the side of the cylinder; if the solid were soft, pressure will be communicated to the sides, but not so much as to the base; if, however, the cylinder is filled with a fluid the pressure will be felt equally at all points of the cylinder. Moreover, the pressure applied at the surface communicates an equal pressure to each point of the cylinder. This is the

Principle of Transmissibility of Fluid Pressure, i.e. **Any pressure applied at the surface of a fluid is transmitted without change of intensity to all parts of the fluid.**

This principle can be experimentally demonstrated as follows:

* For mathematical proof, see Appendix.

Take a vessel of any shape furnished with cylinders opening into it, having air-tight pistons capable of moving in them. Let the vessel be filled with fluid, and forces applied to keep the pistons at rest. Then if the pistons have the same area, and an additional force be applied to one of them, equal forces will have to be applied to the other pistons in order to maintain the state of rest; and if one cylinder has twice the sectional area of the others, twice the force will have to be applied to the piston in it.

The same principle may be also illustrated by means of **the Hydrostatic Press.**

This consists of two vertical cylinders, communicating by means of a pipe, and partially filled with water. Each cylinder is fitted with a piston. Then, if a pressure is applied to the piston A, an equal pressure will be communicated to every point of B; so that if the area of B is ten times that of A, the force of B will be ten times that applied at A; so that by making A small and B large, a small force applied at A can be used to support a heavy weight placed at B.

Ex. Suppose area of piston $A = 12$ square inches, that of B 4 square feet. What force will be required to support 2 tons at B?

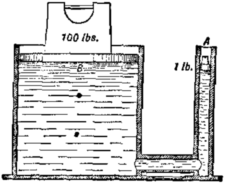
Pressure on $B = 2$ tons weight on 4 square feet;

$= \frac{1}{2}$ ton weight on 1 square foot;

$= \frac{1}{2} \cdot \frac{12}{144}$ ton weight on 12 square inches;

$\therefore$ the force required $= \frac{1}{2} \cdot \frac{12}{144}$ ton weight $= \frac{1}{24}$ ton weight.

The force may be obtained by pouring water into the cylinder A; if the cylinder is long, but very thin, we shall thus have a small quantity of fluid supporting a heavy weight at B; this is known as the Hydrostatic Paradox.



On Pressure at any point of a Fluid at rest under Gravity.—**The Object of Hydrostatics** is to consider the conditions of equilibrium of (i) fluid bodies, and (ii) of solid bodies wholly or partially immersed in fluids.

In all cases considered in this book, the only forces acting on the fluid particles are their own weights and the forces due to the adjacent particles of matter. Now, whatever force the particle A exerts on the particle B, B exerts an equal but opposite force on A; so that when we investigate the conditions of equilibrium of a mass of particles, each one of which is at rest, we may consider the whole mass at rest under the action of its own weight and of the forces on its surface, because the internal forces cancel each other—in

fact, we may regard the mass of fluid just as if it were a solid, and apply the whole theory of statics of a solid body. This is generally expressed by saying that we may imagine the mass of the fluid to be solidified, provided that its density and volume remain unaltered.

If a system of forces act upon a body, it can be resolved into two components—one horizontal, the other, vertical. If the body is at rest, each of these components must be zero.

NOTE.—The vertical through any point is the direction of a plumb-line passing through the point. A horizontal line through a point is a line perpendicular to the vertical which passes through the point; and a horizontal plane through the point is a plane passing through the point perpendicular to the vertical.

The following propositions are deduced from the fundamental property of a fluid, when the only forces acting upon the given mass of fluid are (1) its own weight, (2) the forces at the boundary.

The three fundamental theorems are:

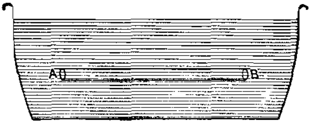
(1) **The pressure is the same at all points in the same horizontal plane.** (PROP. I.)

(2) **The pressure increases uniformly with increase of depth below the free surface.** (PROP. II.)

(3) **The surface of a liquid at rest is horizontal.** (PROP. III.)

PROP. I.—*To prove that the pressure of a liquid at rest is the same at all points of the same horizontal*

Let A and B be two points in a horizontal plane.



Take a thin horizontal cylinder of fluid having its ends at A and B. This cylinder is kept at rest by

(i) The fluid pressures on its ends acting in the direction of the axis.

(ii) The fluid pressures on the curved surface and the weight of the fluid acting perpendicular to the axis.

Since there is no motion in the direction of the axis, the force on the base A = force on the base B.

Let α = area of each end.

Then if p = pressure *at* A; force *on* base $A = p\alpha$.

If p' = pressure *at* B; force *on* base $B = p'\alpha$.

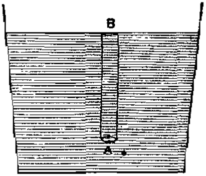
$$\therefore p\alpha = p'\alpha;\ p = p'.$$

Or the pressure is the same at all points in the same horizontal plane.

PROP. II.—*To find the pressure at any point of a liquid at rest, and to prove that it increases uniformly with increase of depth below the free surface.*

Let A be a point in the fluid. On a horizontal area as base, describe a vertical cylinder having its other end in the surface. This cylinder of fluid is kept at rest by

(i) The pressures on its curved surface acting horizontally.



(ii) The force on the base A acting vertically upwards.

(iii) The weight of the fluid acting vertically downwards.

Since there is no vertical motion, the force *on* the base A = weight of the cylinder of fluid.

If a = area of base A, and p = pressure at A, then force *on* the base $A = pa$.

If h = depth of A below the surface, therefore ha = volume of the cylinder.

If w = the weight of unit volume, then wha = weight of the cylinder; therefore $pa = wha$.

$$\therefore p = wh,$$

i.e. the pressure increases uniformly with increase of depth. In this case, where there is no pressure at the surface, the *pressure varies as the depth.*

Cor. I.—In the proof just given we have supposed that there is no pressure on the surface at B. As a matter of fact there will generally be the pressure of the atmosphere on B.

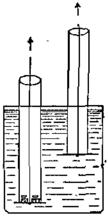
If Π = pressure of the air at any point of the surface, Πa = force of the air on B downwards; and therefore

$$pa = wha + \Pi a, \text{ or } p = wh + \Pi.$$

i.e. the pressure increases uniformly with increase of depth.

COR. II.—In the proposition we prove that the force on the base A is equal to the weight of the cylinder of fluid, *i.e.* is equal to the weight of the fluid vertically above it. If the depth of the area is h feet, then the force on a square foot is the weight of h cubic feet of the fluid. If the depth is h inches, the force on a square inch is equal to the weight of h cubic inches of the fluid. If the atmospheric pressure is taken into account, we must find the force due to the atmosphere upon an equal area, and add it to the weight of the fluid cylinder.

The following **experimental illustration** of this proposition can be given: Take a glass cylinder open at both ends; the glass chimney of an Argand burner will do very well. Grind one end into a true plane. Suspend a ground glass or metal plate from its centre by a string which passes through the cylinder. Keeping the plate close to the cylinder, lower the cylinder into a deep vessel of water until the force of the water is capable of supporting the plate. Now slowly raise the cylinder, and measure the depth of the plate when it begins to fall. It will fall when the weight of the plate is just a little greater than the force of the water upon it. If the area of the base of the cylinder is 12·56 sq. cm. (*i.e.* 4 cm. diameter), and the weight of the plate is 125·6 gmwt., it will be found to fall at a depth of 10 cm. Now increase the weight by placing 125·6 gm. on the plate. The plate will now be found to fall at a depth of 20 cm. So that at a depth of 10 cm. the force of water on the plate is 125·6 gmwt., and at a depth of 20 cm. the force of water on the plate is 125·6 + 125·6 gmwt., *i.e.* on doubling the depth the pressure due to the water is doubled. If the weight is increased by 12·56 gmwt., the depth will be increased by 1 cm.; and if the weight is still further increased by 12·56 gmwt., the depth is still further increased by exactly one centimetre. *So that the force of the water on any plane area increases uniformly with increase of depth, and therefore the pressure or force on the unit area (1 sq. cm. say) increases uniformly with increase of depth.*



This experiment will also give the pressure due to the liquid at any depth. For example, when the depth is 10 cm. the weight is 125·6 gmwt., and the area of the plate is 12·56 sq. cm. Therefore the force of water on 12·56 sq. cm. at a depth of 10 cm. is 125·6 gmwt., *i.e.* pressure is $\frac{125 \cdot 6}{12 \cdot 56}$ or 10 gmwt. per sq. cm.

NOTE 1.—The effect of the pressure of the atmosphere can be neglected in this experiment, because it will exert a downward force on the upper face of the plate, and an upward force on the lower face; and since the areas of the two faces acted upon by the pressure of the air are nearly equal (the difference being due to a part of the upper face of the plate being covered by the end of the chimney), these forces are equal and counterbalance each other.

NOTE 2.—The student must be careful to notice that this *experiment proves that the liquid exerts an upward force on the* lower surface of the plate, and that this force depends only on the depth of the plate below the surface, on the area of the plate, and on the specific gravity of the liquid; in fact, it is equal to the weight of a cylindrical column of the liquid, the base of which has the same area as the plate, and the length of which is equal to the depth of the plate.

NOTE 3.—The height of a liquid above a given point or a given horizontal area is called the "head" of the liquid above the point. Hence the pressure on every square inch of a horizontal area varies directly as the head of the liquid above it. If the head of water is 34 feet, the pressure is $14\frac{3}{4}$ lbwt. per sq. inch. (Ex. 1 below). Hence the pressure is often given in "feet" of water. If the pressure is said to be 34 feet of water, it is meant that the pressure is that which would be due to a head of 34 feet of water, *i.e.* a pressure of $14\frac{3}{4}$ lbwt. per sq. inch. Again, if the pressure is 15 inches of mercury, it is a pressure due to a head of 15 inches of mercury, *i.e.* a pressure which is found from the force on a square inch at a point 15 inches deep in mercury, *i.e.* $7\frac{3}{8}$ lbwt. per sq. inch. (Ex. 2 below.)

Any pressure, however caused, can be expressed in feet of water or inches of mercury. For example, the steam in a boiler if at a *temperature of 120° C. exerts a pressure of 28·8 lbwt. per sq. inch*, and this (see Ex. 4 below) is equivalent to a head of 2·304 × 28·8 feet of water, *i.e.* to 66·3552 feet of water, or to 66·3552 ÷ $13\frac{1}{2}$ feet of mercury, *i.e.* to a head of 59 inches of mercury nearly.

Ex. 1.—Find the pressure at a depth of 34 feet in a fresh-water lake, neglecting pressure of the atmosphere.

The force on a square foot at depth of 34 feet
$= $ weight of fluid vertically above it,
$= $ weight of 34 cubic feet of water,
$\therefore$ Pressure $= 34 \times 1000$ ozwt. per sq. foot $= 34000$ ozwt. per sq. foot.

If we wish to find the pressure in lbwt. per square inch at a depth of 34 feet, we proceed as follows:

The force on a square inch at a depth of 34 feet
$= $ weight of cylinder of fluid $34 \times 12\,(=408)$ inches long,
$= $ weight of 408 cubic inches of water,
$\therefore$ Pressure $= \frac{408}{1728} \times 1000$ ozwt. per sq. inch $= 14\frac{3}{4}$ lbwt. per sq. inch nearly.

Ex. 2.—Find the pressure in lbwt. per square inch at a depth of 30 inches in mercury. (Specific gravity $= 13\frac{1}{2}$.)

The force on a square inch at a depth of 30 inches
$= $ weight of 30 cubic inches of mercury,
$= 13\frac{1}{2} \times \frac{30}{1728} \times 1000$ ozwt.
$\therefore$ Pressure $= 14\frac{83}{128}$ lbwt. per sq. inch $= 14\frac{3}{4}$ lbwt. per sq. inch nearly.

Ex. 3.—A cistern is 14 feet long, 12 feet broad, and 8 feet deep. Find the force on the base when the cistern is full of water, the pressure of the atmosphere being $14\frac{3}{4}$ lbwt. per square inch.

The force on the base $= $ weight of water above it $+$ force of lbwt. on every square inch of the base.

The weight of the water $= $ weight of $14 \times 12 \times 8$ cubic feet of er $= 14 \times 12 \times 8 \times 1000$ ozwt.

Area of base $= 14 \times 12$ square feet $= 168$ square feet
$= 168 \times 144$ square inches.

$\therefore$ force of $14\frac{3}{4}$ lbwt. on every square inch of base gives a force of $168 \times 144 \times 14\frac{3}{4}$ lbwt. on the whole base.

force on the base
$= 14 \times 12 \times 8 \times 1000$ ozwt. $+ 168 \times 144 \times 14\frac{3}{4}$ lbwt.
$= 84000$ lbwt. $+ 356832$ lbwt.
$= 4408$[illegible] lbwt.

Ex. 4.—Find the head of water which gives a pressure of 1 lbwt. per sq. inch.

The question is, what is the length of a cylinder of water of 1 sq. inch sectional area which will weigh 1 lb.?

Now, 1000 oz. is the weight of 1728 cubic inches.

1 oz. „ „ $\frac{1728}{1000}$ „

1 lb. „ „ $\frac{16 \times 1728}{1000}$ cubic inches.

i.e. 27·648 cubic inches.

Now, a cylinder 27·648 inches long and 1 square inch sectional area will have a volume of 27·648 cubic inches. Hence the pressure at the depth of 27·648 inches in water will be 1 lbwt. per sq. inch.

Therefore a pressure of 1 lbwt. per sq. inch is equivalent to a head of 27·648 inches (*i.e.* 2·304 feet) of water.

EXERCISES.

1. Find the pressure in lbwt. per square inch at a depth of 100 feet in water, and at a depth of 30 inches in mercury.

2. Find the force on a horizontal plane of 5 square feet placed 25 feet deep in glycerine (sp. gr. 1·25).

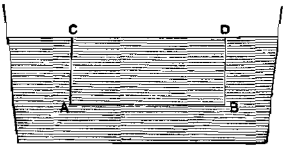
3. Find the depth to which a plane of 3 square inches area must be sunk in alcohol (sp. gr. ·85) so that the force upon it shall be 13 *lbwt.*

4. Find the force on the horizontal base of a vessel filled to a height of 10·5 cm. with mercury, the base of the vessel being rectangular and 9 cm. long by 6 cm. broad, the pressure of the atmosphere being equal to the weight of 76 cc. of mercury per sq. cm.

5. Find the head of mercury which will give a pressure of 15 lbwt. per sq. inch.

6. Find the force on an area of 24 square feet due to a

Prop. III.—*The surface of a liquid at rest is horizontal.*



Take two points A and B in a horizontal plane in the fluid. Join AB and draw AC, BD vertical to meet the surface of the fluid in C and D.

Since A and B are in a horizontal plane,

pressure at A = pressure at B (Prop. I.).

$\therefore w.CA = w.BD$ (Prop. II.), $\therefore CA = BD$.

Since CA is both equal and parallel to BD, the line CD is equal and parallel to AB. But AB is horizontal, and therefore CD is horizontal. Hence the line joining any two points of the surface is horizontal.

Note.—The surface is horizontal when the pressure of the atmosphere is taken into account.

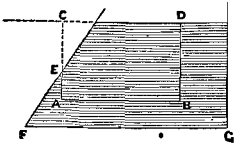
$$\text{For pressure at } A = \Pi + w.CA;$$
$$\text{pressure at } B = \Pi + w.BD;$$
$$\text{and, as before, } w.CA = w.BD;$$
$$CA = BD.$$

Cor. I.—*The pressure at a point of fluid depends only on the depth below the surface of the fluid.*

Thus in the vessel the pressure at A = pressure at B at same level (Prop. I.); and pressure at $B = w.BD$

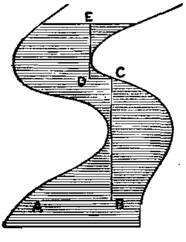
(PROP. II.). Therefore pressure at $A = w . BD$, and $BD = AC$.

Therefore the pressure at A depends only on the



depth of A below the surface CD. The reason for this is that, although the height of the column above A is only AE, there is a downward pressure at E on this column equal to the weight of a column CE.

This proof may be extended to the more general case of the pressure at any point of a liquid in a vessel of any shape.



Let A be a point in a liquid.

Draw $ABCDE$ to meet the surface in E, so that AB, CD are horizontal; BC, DE vertical.

Now,

pressure at C = pressure at D

$= w . DE$.

Pressure at A = pressure at B.

$= w . BC +$ pressure at

$= w . BC + w . DE.$

$= w \times$ depth of A below the surface.

'. the pressure at A varies as the depth below the ce.

Cor. II.—*The force on the base of a vessel is independent of the shape of the vessel.*

For the force on every square inch of the base FG s equal to the weight of cylinder of fluid, whose base is a square inch, and height of which is equal to the depth of the base below the surface of the fluid. Hence the orce on the base is equal to the weight of the cylinder of fluid, having for its base the base of the vessel, and or its height the depth of the base below the surface of the fluid.

Hence the force on the base of a vessel containing a liquid may be equal, greater, or less than the weight of the liquid in the vessel, according as the sides of the vessel are vertical, or slope inwards (as Fig. 1, Cor. I.) or outwards (as Fig. Prop. III.). When the force s greater than the weight of the liquid, the sides of the vessel loping inwards exert at each point (as at E, Fig. 1, Cor. I.) a lownward pressure to prevent the liquid rising, and the pressure is ransmitted to the base. The pressure at the point of the base vertically below E is due, not only to the height of E above the base, but also to the pressure at E, which is that due to the depth of E below the surface. When the force on the base is less than the weight of water in the vessel, then the part of the water which is not vertically above the base is supported by the outward sloping sides of the vessel.

Ex.—Suppose a conical vessel filled with water and closed with a lid, and then inverted on a table. Find the force of the fluid on the base, and the force on the table if the

eight of the cone is 6 feet and the area of base 2 square ɜet.

Force of the fluid on the base = weight of cylinder of water, ɪe base of which is 2 square feet and height 6 feet.

= weight of 6 × 2 cubic feet = 12 × 1000 ozwt. = 12000 ozwt.

Force on table = weight of cone + weight of water it contains.

Now, the volume of a cone is one-third that of a cylinder of ɋual base and altitude.

$\therefore$ volume of cone $= \frac{1}{3} . 12$ cubic feet = 4 cubic feet.

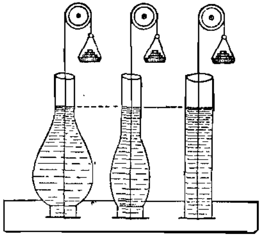
$\therefore$ weight of water in the cone = 4000 ozwt.

$\therefore$ Force on the table = weight of cone + 4000 ozwt.

That the force which a liquid exerts on the base of a essel is independent of the shape of the vessel, but epends only on the area of the base and the depth of ɪe liquid, can be proved by the following experiment:

Take a glass vessel of any shape (say an ordinary lamp chimney), ɪd grind one end flat. Fix the vessel in a retort stand. Form its ɪse by a ground piece of glass or a metal plate, having a string ;tached so that the plate will hang horizontally. Pass this string ɪrough the vessel, and suspend the moveable base from one end of balance, or pass the string over a pulley. Now place weights in ɪe scale-pan just to balance the base, and then place an additional eight in the scale-pan. Adjust the vessel so that the beam is ɪrizontal when the base is in its position at the bottom of the ɜssel, and so that the base is horizontal and the string vertical. ow pour water into the vessel, and by means of a vertical scale ɔtice the height of the water when the base begins to fall. Now, ɪe base will begin to fall when the force on it is equal to the ɪditional weight. Therefore the additional weight will be the force ɪ the base at the observed height. It will be found that whatever ɪaped vessel be taken, the force is equal to the weight of water hich would be contained in a cylinder the base of which is equal to ɪe base of the vessel, and the length of which is equal to the depth the water. For example, two chimneys were found, the bases of hich were of equal area 12·56 sq. cm. (*i.e.* 4 cm. diameter). In ɪch case an additional weight of 125·6 gm. was placed in the scale, ɪd it was found that although one was a true cylinder and the ɪher bulge-shaped, the base fell at a depth of 10 cm. in each. For each case area of base was 12·56 sq. cm., depth 10 cm.; and hence

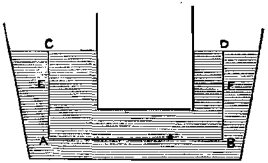
volume of water vertically above the base 125·6 cc. and its weight 125·6 gmwt.



Another instructive experiment is to make the top of the vessel water-tight by means of a cork, and pass a narrow piece of glass tubing through the cork; then pour water into the vessel until it rises into the narrow tubing. It will be found that the force on the base still depends simply on the area of the base and the depth of water or the head. Take a bladder, and fix a long narrow piece of glass tubing to it, and fill it with water. By pouring a small quantity of water into the tube the bladder may easily be burst if the tube is held vertically. In order to save it, incline the tube, thereby diminishing the head of water above each part of the bladder.

Whatever pressure is added to the pressure at the surface is transmitted without change to all parts of the liquid. If the vessel is a cylinder, and fitted with a water-tight piston, and the piston loaded, the pressure is transmitted to all parts of the cylinder. The tube in the above experiment could be fitted with a piston, and then the pressure could be increased by placing weights on the piston. This is the principle of the hydraulic **accumulator**, which consists of a strong iron cylinder filled with water, within which a ram moves through a water-tight collar. The ram is loaded with a considerable weight, and a pressure of 700 lbwt. per square inch can easily be obtained. This pressure can be applied at any point, however distant, by connecting the point to the accumulator by a strong iron pipe.

COR. III.—The proof in PROP. III. holds equally good when the line CD does not lie entirely in the surface of the liquid, *i.e.* when there is a solid between



C and D, as in the figure, provided that AB lies in the liquid. It is also true even if the verticals AC, BD do not lie entirely in the liquid, for by COR. I. the pressure at A is still equal to $w \, . \, AC$, and pressure at $B = w \, . \, BD$.

The fact that the surfaces of a liquid, on which the surface pressure is the same at all points, are horizontal is expressed by saying that **liquids maintain their level.** Thus, if a number of pipes lead from a reservoir, the water will rise in each of them to the height of the surface of the water in the reservoir; so that if no part of a pipe be above this level, water can be drawn from the reservoir by means of it at any point below this surface. Reservoirs are therefore generally formed in elevated positions.

COR. IV.—*The pressure is the same at all points of a horizontal plane in a liquid, even when the lines joining the points do not lie entirely in the liquid.*

Let E, F be in the same horizontal plane.

The pressure at $E = w\,.\,CE$.

The pressure at $F = w\,.\,DF$.

And CD is horizontal, and therefore parallel to EF.

$\therefore\ CE = DF$.

And therefore pressure at E = pressure at F.

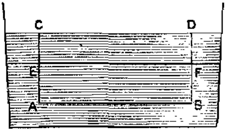
An important practical use is made of this fact in levelling large tracts of land. Two vessels fixed upon separate supports are connected by a long pipe, and then filled with water. The surface of the water in each vessel will be at the same level; and therefore two points fixed respectively at exactly the same vertical distance above each surface will be at the same level, and hence, by sighting these two points, a point or series of points can be marked off which are in the same horizontal plane.

A smaller level consists of a glass cylinder filled with alcohol, with the exception of a small bubble of air which is left to form the index. The air bubble will always occupy the highest position. The glass cylinder is either worked into a true plane on one side, or is fixed in a case which has one side a true plane, or rests upon two supports. To test if a surface is horizontal the level is placed on the surface and the position of the bubble observed. The level is then reversed (turned round two right angles), and the position of the bubble observed. If it coincides with its first position, then the line on which the level is placed is horizontal. In the same way it is found if the line at right angles to this is level. If both lines are level, then the surface is horizontal. The plane surface of the level case is arranged so that the centre of the bubble is at a mark near the centre of the tube when the tested surface is horizontal.

PROP. IV.—*Two liquids which do not mix are placed in a vessel. To find the pressure at a given depth in the lower liquid.*

Let FE be the common surface of the two liquids. Let A be a point in the lower liquid. Draw AEC vertical to meet FE in E and the free surface in C.

If w = weight of unit volume of the upper liquid, then the pressure at $E = w \cdot CE$.



If w' = weight of unit volume of the lower liquid, the pressure at $A = w' \cdot EA$ + pressure at E;
$$= w' \cdot EA + w \cdot CE.$$

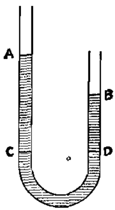
COR.—*The common surface of two liquids which do not mix is horizontal.*

Take two points A, B in a horizontal plane in the lower liquid. Draw BFD vertically to meet FE in F and the free surface in D.

Now, the pressure at A = pressure at B (PROP. I.).
$$\therefore w' \cdot EA + w \cdot CE = w' \cdot BF + w \cdot FD \ldots\ldots(\text{i}).$$
Since DC and BA are parallel, $AC = BD$,
$$\therefore EA + CE = BF + FD.$$
Therefore multiplying by w',
$$w' \cdot EA + w' \cdot CE = w' \cdot BF + w' \cdot FD,$$
and subtracting from (i)
$$CE(w - w') = FD(w - w'); \quad \therefore CE = FD,$$
and therefore FE is horizontal.

Ex.—Two liquids whose specific gravities are s and s' are poured into a bent tube. Find the ratio of the heights of their surfaces above the common surface.

Let A, B be their free surface, C their common surface, and D in same horizontal plane as C.



Let s = sp. gr. of fluid in AC;

s' = sp. gr. of fluid in BDC,

then pressure at C = pressure at D.

$$\therefore s \cdot AC = s' \cdot BD;$$

$$\therefore \frac{AC}{BD} = \frac{s'}{s}.$$

or $AC : BD :: s' : s$.

Thus, if the liquid in BC is mercury, and that in AC water, then since the specific gravity of mercury is $13\frac{1}{2}$, $AC = 13\frac{1}{2}BD$.

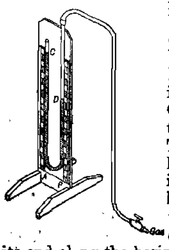
If the height or head BD is one inch, then the corresponding head of water is $13\frac{1}{2}$ inches; if the head BD is increased by an inch of mercury, then the head of AC must be increased by $13\frac{1}{2}$ inches of water. Now, the effective head BD can be increased without adding more mercury. A piston just fitting the tube can be placed on the surface at B, and loaded with a weight which will produce an equal pressure. Suppose the area of the tube at D is 1 square inch, and a piston be fitted and loaded so as to produce a total weight of 1 lb. Then the pressure is 1 lbwt. per square inch. This corresponds to 2·304 feet of water, *i.e.* to 26·448 inches of water; hence the corresponding height of AC will be 26·448 inches. Hence the head of water above the level of D will give the pressure at D. In fact, an instrument for measuring the pressures at different points of a fluid is made on this principle, and is called a **pressure gauge.** For example: The pressure of the atmosphere is measured by a pressure gauge. If the U tube contains only mercury, and the branch B be joined to a vessel containing a gas, the pressure on the surface B will be measured by the head of the mercury in the branch A above the level of the mercury in B. If the head is zero, *i.e.* if the surface of the mercury is at the same level in both branches, then the pressure on the surface at B = the pressure on the surface

at A, *i.e.* if A is open to the atmosphere, it is equal to the pressure of the atmosphere. If the head is 3 inches of mercury, then the pressure of the gas at B is greater than the pressure of the atmosphere by 3 inches of mercury; and if the pressure of the atmosphere corresponds to 30 inches of mercury, then the pressure of the air in B is 33 inches of mercury. The pressure of wind can also be measured by a water gauge, one branch of which is open towards the wind, the other being protected from the wind.

EXPERIMENTS ON CHAPTER III.

EXP. I.—*To find the pressure of the gas in the laboratory.*

Make a gauge in the following manner: Bend a piece of narrow glass tubing into a U tube, with the arms parallel to each other, and about 18 inches long. Take a board 20 inches by 4 inches, and nail two pieces of wood to one end at right angles to the board to form feet for it. Fix the tube on the board with its arms vertical, and draw a horizontal line AB on the board near the bottom of it. Pour water into the tube so that it stands 6 inches in each arm. Connect one arm by india rubber tubing to the gas connexion. Turn on the gas. Measure the heights AC, BD of the surfaces in each arm above the horizontal line. To do this place the half-metre scale with its graduated edge close to the tube, and with its end along the horizontal line. Now placing the eye a little distance off and on a level with the surface of



the water, read the height of the water in the arm. The difference between the two heights in millimetres will give the difference between the pressure of the gas and the pressure of the atmosphere in mm. of water.

Also find the difference of pressure in mm. of alcohol, turpentine, and glycerine.

Ex. 1.—Height of water in open branch 17·9 cm.
Height of water in gas branch 11·2 cm.
Difference = 6·7 cm. = 67 mm.

∴ the pressure of the gas is greater than that of the atmosphere by 67 mm. of water.

Ex. 2.—Height of alcohol in open branch 17·9 cm.
Height of alcohol in gas branch 10·0 cm.
Difference = 7·9 cm. = 79 mm.

∴ the pressure of the gas is greater than that of the atmosphere by 79 mm. of alcohol.

Test the truth of these two measurements by finding the specific gravity of alcohol.

79 mm. of alcohol is equivalent to 67 mm. of water.
1 mm. " " $\frac{67}{79}$ mm. of water.
∴ sp. gr. of alcohol $= \frac{67}{79} = \cdot 848$.

EXPERIMENTAL EXERCISES.

(1) Heights of glycerine surfaces 18·6 cm., 13·1 cm.; of turpentine 19·85 cm., 12·0 cm. Find the pressure of the gas in cm. of water (sp. gr. of glycerine = 1·25, of turpentine ·85).

(2) A U tube is given to you, with one end closed. Pour mercury into the tube until the pressure is greater than that of the atmosphere by 300 mm. of mercury.

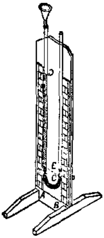
(3) A glass bulb containing air is given to you, fitted with a siphon gauge. Find the pressure of the air inside at 15° C., 30° C., 60° C., and 100° C. respectively.

(4) Find by a mercury gauge the pressure at depths of 5, 10, and 15 cm. respectively in water; and prove that it varies as the depths below the surface.

EXP. II.—*To find by means of a U tube the specific gravity of two liquids which do not mix.*

Use the gauge of Exp. I.

Pour mercury into the tube until it stands about two inches high in each tube. Now pour water into one arm. Measure the head of water and of the mercury above the common level. This can be done by measuring



(1) Height *BC* of common surface above (or below) the fixed horizontal line.

(2) Height of surface of water *AD*.

(3) Height of surface of mercury *BE*.

The difference between (1) and (2) is the head of water *CD*.

The difference between (1) and (3) is the head of mercury *CE*.

Three such experiments should be made with different heads.

Ex.—(1) Height of common surface above line 5·0 cm.
(2) Height of surface of mercury 8·9 cm.
(3) Height of surface of water 57·9 cm.

Calculation.

Head of mercury $8 \cdot 9 - 5 = 3 \cdot 9$ cm.
Head of water $57 \cdot 9 - 5 = 52 \cdot 9$ cm.

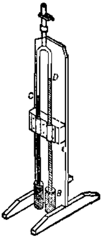
$$\therefore \text{sp. gr. of mercury} = \frac{52 \cdot 9}{3 \cdot 9} = 13 \cdot 56.$$

Ex.—(1) Depth of common surface $= 6 \cdot 75$ cm.
(2) Depth of mercury surface $= 1 \cdot 62$ cm.
(3) Height of glycerine surface $= 48 \cdot 95$ cm.

Find sp. gr. of glycerine.

EXP. III.—*To find the specific gravity of a li*
by the inv

Take a T piece of glass tubing with the horizontal part of the T about 24 inches long, and the vertical part about 2 or 3 inches long. Bend the long tube into a U tube with the arms parallel and equal. Attach a piece of india rubber tubing to the cross-piece. Fix the tube inverted on a board, as in Exp. I., but at a distance of an inch from the board (or it could be supported by a vertical rod). Fill two small beakers, one with water and the other with liquid, and place the open ends of the U tube into them. Now apply the mouth to the india rubber tubing, and draw the liquids up the tube until one of them nearly reaches the top. Now measure the head of each liquid above the *surface of the liquid* in each vessel respectively.



Example.

Height of water AC in one branch of tube 28·35 cm.
Height of alcohol BD in the other branch 33·3 cm.

$$\therefore \text{sp. gr. of alcohol} = \frac{28 \cdot 35}{33 \cdot 3} = \cdot 851.$$

EXAMPLES ON CHAP. III.

1. Find the pressure in lbwt. per sq. inch at a depth of 0 inches in water, in mercury, and in alcohol.

2. If the head of water above a point is 300 feet, what is the pressure in lbwt. per sq. inch?

3. Find the pressure in lbwt. per sq. inch due to a head of 1 inch of mercury and to 1 foot of water.

4. What head of water is equivalent to a pressure of 4·7 lbwt. per sq. inch?

5. What head of mercury is equivalent to a pressure of 14·7 lbwt. per sq. inch?

6. If the head of water is one foot, what is the pressure in (1) lbwt. per sq. foot; (2) lbwt. per sq. inch; (3) in atmospheres; (4) in inches of mercury (*i.e.* corresponding head in mercury)?

7. Find the heads (1) in water, (2) in mercury corresponding to the following pressures: 1 lbwt. per sq. foot; 1 lbwt. per sq. inch.

8. Find the pressure in gmwt. per sq. cm. at a depth of 760 mm. in mercury, and at a depth of 1 cm. in mercury.

9. Find the head in inches of water of 76 cm. in mercury.

10. Find the pressure in poundals per sq. foot of a head of 29·922 inches of mercury.

11. Find the pressure in dynes per sq. cm. of a head of 76 cm. in mercury.

12. A cylinder 2 feet in diameter is fitted with a piston and filled with water. A weight of 3 tons is placed on the piston. Find the pressure in lbwt. per sq. inch.

13. An area of 2 sq. inches is divided into 100 equal parts. The force on each part is approximately the same, and is $\frac{1}{10}$ ozwt. Find the pressure at each point in lbwt. per square foot.

14. Find the force on a circular ram 18″ in diameter which works in a cylinder supplied with water from a tank 174 feet above the base of the ram.

15. The ram of a hydraulic accumulator is 10 inches in diameter, and is loaded with 30 tons. Find the force on a square inch.

16. What load must be placed on a ram 7·5 inches in diameter to produce a pressure of 700 lbwt. per sq. inch?

17. Find the force on a ram of 10 inches diameter due to a load of 10 lbwt. on an area of $\frac{1}{4}$ sq. inch.

18. A U tube has mercury poured into it until the surface of the mercury is half way up each tube. Water is poured into one branch until it is filled. How high will the mercury rise in the other branch?

19. A U tube, having its two arms equal, has mercury poured into it until each surface is 6 inches from the top of the tube. Water is now poured into one branch and alcohol into the other so as to fill the tube. Find the length of the tube occupied by each liquid.

20. A U tube has water poured into it, and then oil is poured into one arm. If the sp. gr. of the oil is ·65, what length of tube must it occupy to alter the level of the water by *3 inches?*

21. Upon what does the pressure at any point of a liquid at rest under the action of its own weight depend?

22. If the pressure at a depth of 6 feet is 5 lbwt. per sq. inch, what is the pressure at a depth of 7 feet?

23. The pressure at a depth of 30 inches is $14\frac{3}{4}$ lbwt. on a square inch. What is the pressure at a depth of 46 feet on (1) a square inch; (2) a square yard?

24. If the pressure at a depth of 5 fathoms is 15 lbwt. per sq. inch, what is the pressure at a depth of 2 inches?

25. Find the pressure on a square foot at a depth of 10 feet in mercury. (Specific gravity = 13·5.)

26. The pressure at a depth of 2 feet in a liquid is 3848 lbwt. on a square foot, and at a depth of 3 feet it is 4692 lbwt. Find the specific gravity of the liquid and the pressure at the surface.

27. If a cubic foot of water weighs 1000 oz., what is the pressure on a square inch at the depth of a mile below the surface of the sea?

28. The pressure at the bottom of a well 61 feet deep is $2\frac{1}{2}$ times the pressure at 7 feet below the surface. Find the pressure of the atmosphere on a square inch at the

29. If the pressure at the bottom of a well is 4 times that at the depth of 2 feet, what is the depth of the well if the pressure of the atmosphere is equivalent to 30 feet of water?

30. Compare the pressures at points 15 and 60 inches respectively below the surface of a basin of mercury, taking into account the pressure of the atmosphere, which is $14\frac{3}{4}$ lbwt. on a square inch.

31. The pressure at a point 4 feet below the surface of a heavy fluid is 30 lbwt. per sq. inch, and at a depth of 9 feet it is 50 lbwt. per sq. inch. What is the pressure at the surface?

32. What is the pressure per square inch at the bottom of a well 23 feet deep, the pressure of the atmosphere on a square inch being equal to the weight of a column of mercury 31 inches high vertical above it?

33. If the atmospheric pressure at the surface of the earth be 14 lbwt. on the square inch, and the height of a column of air of the same uniform density as at the surface which would produce the same pressure, be 5 miles, show that the density of air is $\frac{1}{800}$ nearly, the density of water being 1 and the weight of a cubic foot of water 1000 oz.

34. When a flat-bottomed vessel contains water, how is the force on the base determined? Under what circumstances is this the same as the weight of water in the vessel? When it is not so, what other forces make up the difference?

35. A bowl containing water is placed on a table; the weight of the bowl is 6 oz., the area of its base is 28 square inches, and the depth of the water 5 inches. Supposing the bowl to be of such a shape that only three-fourths of the fluid is vertically over the base, determine the pressure at any point of the base, the whole pressure on the base, and the pressure on the table.

36. The pressure in the water-pipe at the basement of a building is 34 lbwt. to the square inch, whereas at the third floor it is only 18 lbwt. to the square inch. Find the height of the third floor.

37. A hollow cone stands with its base on a horizontal table. The area of the base is 100 square inches, and the height 8·64 inches; its weight is equal to the weight of water it will contain. When filled with water, what is the ratio of the force of the water on the base to that of the base on the table? How do you account for the result? [The volume of a cone is one-third of that of a cylinder with the same base and altitude.]

38. A cubical box whose edges measure 1 foot has a pipe communicating with it which rises to a vertical height above the lid of 20 feet. It is filled with water to the top of the pipe. Find the upward force on the lid and the downward force on the base, and show that their difference is equal to the weight of water in the box. How do you account for the force on the base being greater than the weight of water it has to support?

39. The two branches of a uniform bent tube are straight and vertical, and the portion of the tube which unites them is horizontal. Water is poured in sufficient to fill 6 inches of the tube; and then oil, sufficient to occupy 5 inches, is poured in at one end, the specific gravity of oil being four-fifths that of water. Find the position of the fluids when they are in equilibrium, the horizontal part of the tube being 2 inches long.

40. A vessel, whose base is a square the side of which measures 6 inches, contains mercury to the depth of an inch, and water is poured upon the mercury to the depth of $10\frac{1}{2}$ inches. If the specific gravity of mercury be 13·5 times that of water, find the force on the base of the vessel.

41. A tin vessel shaped like a pyramid on a square base of 12 inches edge is filled with water; the pyramid is 18 inches high. What is the force on the base?

42. Two vessels with square horizontal bases, whose edges are 10 inches and 9 inches respectively, are full of water. If the forces on their bases are in the ratio of 20 : 9, compare the depths of the fluid in the vessels.

43. Two vessels, whose bases contain respectively 12 and 14 square inches, are filled with water, the first to a height of 6 inches, the second to a height of 10 inches. Compare the forces on their bases, and the pressure at any point of their bases.

44. Water is poured into a U tube, the legs of which are 6 inches long, until they are half full. As much oil as possible is then poured into one of the legs. What length of the tube does it occupy, water being half as heavy again as oil?

45. Two rectangular cisterns standing on a horizontal plane are joined at their bases by a leathern pipe resting on the plane. If one of them be 5 ft. long and 3 ft. broad, and the other 4 ft. 6 in. long and 3 ft. 4 in. broad, and water be poured into either, then, when the water is at rest, the forces on the bases will be equal.

46. Three rectangular cisterns are filled with water. One of them is in the form of a cube, whose edge is 8 ft.; another is 5 ft. high, 5 ft. wide, and 14 ft. long; and the third is 3 ft. high, 3 ft. wide, and 18 ft. long. Show that the force on the base of the first is 14 tons 640 lbwt.; and that this force is equal to the sum of the forces on the bases of the other two, the atmospheric pressure not being taken into consideration.

47. Compare the pressures at any points on the base of each cistern in the preceding example.

CHAPTER IV.

ON THE EQUILIBRIUM OF SOLID BODIES IMMERSED IN A FLUID.

If a body be placed in a liquid, every small area of the surface of the body in contact with the liquid is acted on by a force which, as we have seen in the previous chapters, varies as the depth of the area below the surface of the liquid, and the direction of which is perpendicular to the small area. Now, the body is at rest under the action of these forces, and of the other forces which may act upon it, such as its own weight, or the pull of strings.

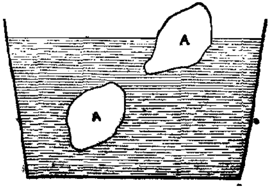
Before considering the equilibrium of a body immersed in a liquid, it is necessary therefore to find the resultant force of the fluid upon it. In this investigation it is only necessary to consider the part of the body below the surface of the liquid, for this is the only part subject to fluid pressure. Since the pressure at any point of a liquid depends only on the depth of the point below the surface, and on the specific gravity of the liquid, the resultant force of a fluid on a body *will be independent of the substance of the body.* Thus, if a rod be held in water, the pressure of the water on the rod will be just the same if the part of the rod immersed be made of wood, or iron, or sealing-wax, or even water. The difference between the several cases is this, that

whereas in the case of wood, or iron, or sealing-wax, the rod would generally require to be held in position; in the case of the water the part considered, *i.e.* the part below the surface, would always be at rest without the action of external forces, except its own weight and the fluid pressures.

In order to find the *resultant force which a fluid exerts on a body immersed in it,* we can imagine the part of the body below the surface to consist of fluid of the same density, and consider its equilibrium under the action of its own weight and the fluid pressures. The weight of this liquid is equal to the weight of the fluid actually displaced by the solid, and the centre of gravity of this liquid is called the centre of buoyancy, or the centre of gravity of the fluid displaced.

The student will now be in a position to understand the following **general proposition** in the theory of the equilibrium of solids immersed in a liquid:

To find the resultant force which a fluid exerts upon a body totally or partly immersed in it.



Let A be the body totally or partly immersed in a liquid. Since the fluid pressure on the body is independent of the substance of the body, we can imagine the substance of the body below the surface to be liquid. But this liquid would be at rest, and the forces keeping it at rest would be

(1) Its own weight acting vertically downwards through the centre of gravity of the liquid.

(2) The resultant force of the fluid.

And these must be equal.

Therefore this resultant force must (1) be equal to the weight of the liquid displaced; (2) act vertically upwards; (3) pass through the centre of gravity of the liquid displaced; and this resultant force is the same as that on the body.

NOTE.—It is especially to be noticed in this proposition that the body may be maintained at rest by any force whatsoever. The consideration of the equilibrium of the *body* forms the subject of the following propositions. If the only force besides that due to the fluid, acting on the body is its weight, the body is said to float freely.

PROP. I.—*To find the conditions of equilibrium of a body floating freely in a liquid.*

Let A [in preceding figure] be the body floating *freely.*

The body is kept at rest by

(1) Its own weight acting vertically downwards through the centre of gravity of the body.

(2) The resultant force of the fluid.

Therefore the weight of the body = resultant force of the fluid.

But the resultant force of the fluid = weight of fluid displaced by the body.

Therefore (i) the weight of the body = weight of fluid displaced.

Since the only forces acting on the body are its weight and the resultant force of the fluid, their directions must be coincident.

Therefore (ii) the line joining the centres of gravity of the body and of the fluid displaced must be vertical.

Cor.—If a body float freely in a liquid, its volume is to the volume immersed as the specific gravity of the fluid is to the specific gravity of the body.

For if V cubic feet = volume of body, and S its specific gravity, then weight of body $= V.S.1000$ ozwt.

If V' cubic feet = volume of part immersed, S' = specific gravity of liquid, then

weight of fluid displaced $= V'.S'.1000$ ozwt.

But these are equal.

$$\therefore V.S = V'.S', \text{ or } \frac{V}{V'} = \frac{S'}{S}.$$

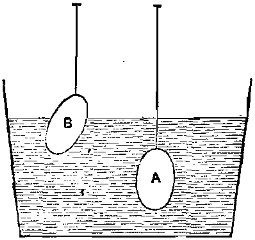
If $V' = V$, *i.e.* if the body floats entirely immersed, then $S' = S$, *i.e.* the specific gravity of the body and fluid must be equal.

Prop. II.—*To find the conditions of equilibrium of a body floating in a liquid and partly supported by a string, the pull of the string acting upwards.*

The body is kept at rest by

(1) Its own weight acting vertically *downwards* through the centre of gravity of the body

(2) The resultant force of the fluid, which is equal to the weight of fluid displaced, and acts vertically



upwards through the centre of gravity of the fluid displaced.

(3) The pull of the string acting *upwards*.

∵ pull of string + weight of fluid displaced = weight of body.

∴ pull of string = weight of body − weight of fluid displaced.

Since the directions of the weight of the body and of the weight of fluid displaced are vertical, the direction of the string must be vertical.

COR.—If V, V', S, S' have the same meaning as in the last proposition, and T = pull of the string,

$$T = V.S.1000 \text{ ozwt.} - V'.S'.1000 \text{ ozwt.}$$

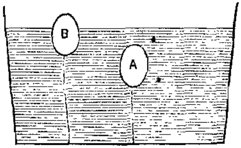
$$= (V.S - V'.S') \times 1000 \text{ ozwt.}$$

When the body is totally immersed,

V and $T = V(S - S')\, 1000$ ozwt.

NOTE.—If the body is totally immersed, the centres of gravity of the body and of the fluid displaced are coincident, and therefore the directions of the string, the weight of the body, and the weight of fluid displaced are coincident. If the body is partly immersed, these directions will not generally be coincident.

PROP. III.—*To find the conditions of equilibrium of a body floating in a liquid and partly supported by a string, the pull of the string acting downwards.*



In this case the condition of equilibrium is

Pull of string + weight of body = weight of fluid displaced.

Pull of string = weight of fluid displaced − weight of body.

COR. $T = (V'.S' - V.S)$ 1000 ozwt.

The *general* proposition was first proved by Archimedes (a Greek philosopher born in 287 B.C., just after the death of Euclid), and is known as the principle of Archimedes. It is of the greatest importance, and should be thoroughly understood.

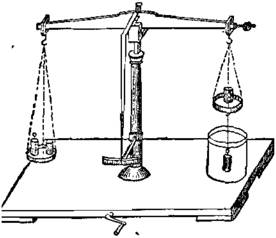
The **principle of Archimedes** gives the *magnitude*, *direction*, and *position* of the force which a fluid exerts upon a body immersed in it. It states that this force

(1) Is equal to the weight of the fluid displaced by the part of the body below the surface.

(2) Acts vertically upwards.

(3) Acts through the centre of gravity of the fluid displaced.

The following **experimental verification of the principle** will make it clear: Take a solid brass cylinder, and either make of card-board, or better, of brass tubing, a cylindrical vessel into which the cylinder will exactly fit, so that the volume of the solid cylinder equals the internal volume of the vessel. Suspend the solid from the hook of the scale-pan in a hydrostatic balance, and place



the vessel in the same scale-pan. Now balance these two by weights placed in the other scale-pan. Place a vessel containing water so that the solid is immersed in the liquid. The solid will now appear lighter, because it is supported by the water, and it will require a downward force in order to keep the solid totally immersed, and to overcome the upward thrust of the water. Fill the hollow vessel carefully with water up

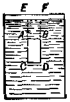
to the brim. It will be found that if the solid is totally immersed, equilibrium will be restored. Therefore the weight of water which fills the vessel has balanced the upward thrust of the water. Therefore the upward thrust, or buoyancy, of the water is a force equal to the weight of the water displaced by the solid, because the weight of water contained by the cylindrical vessel is just equal to the weight of water the solid displaces.

If the vessel be half filled with water, it will be found that equilibrium is obtained when the solid is just half immersed. Therefore the buoyancy of the water is equal to the weight of water displaced.

(Other methods of proving the principle experimentally are given at the end of the Chapter).

NOTE.—It will be observed that the buoyancy of the liquid on the body does not depend upon the depth the body is immersed, provided that the specific gravity of the liquid does not vary. The reason for this can be easily explained in the case of a cylindrical body with its axis vertical.

Let $ABCD$ be such a body immersed in a liquid. Since the liquid does not exert any vertical force on the vertical sides of the body, the vertical forces acting on it are (1) the force of the liquid acting downwards on the upper surface AB, which is equal to the weight of the liquid column $ABEF$; (2) the force of the liquid acting upwards on CD, which is equal to the weight of the liquid which would be contained in a column $CDEF$. Hence the resultant force of the liquid is equal to the difference of these weights, *i.e.* to the weight of the liquid displaced by $ABCD$. If the body is depressed (say) a foot, the downward force on AB is increased, but the upward force on CD is increased by an equal amount, and so the resultant force on the body is unaltered.



If a rectangular block of pine wood be floated in water, it will be found that about half of it is immersed. The volume of the block can be found by measurement, and hence the volume of water displaced. If the block of wood be weighed, it will be found equal to the weight of water displaced (which is an experimental

verification of PROP. I.). Hence the weight of the wood is equal to half the weight of an equal volume of water. This shows that the specific gravity of the wood is $\frac{1}{2}$. Iron is $7\frac{1}{2}$ times as heavy as water, and mercury is $13\frac{1}{2}$ times as heavy as water; therefore mercury is $13\frac{1}{2} \div 7\frac{1}{2}$, *i.e.* 1·8 times as heavy as iron. If for simplicity of explanation we regard mercury as twice as heavy as iron, we should expect that iron would float in mercury with half its volume immersed. Floating an iron cylinder in a basin of mercury, it is seen to be about half immersed. In order to totally immerse the iron in mercury a downward force would have to be exerted upon it equal to the weight of the additional mercury which would be displaced.

If a rectangular box, made water-tight, be floated in *water, it will sink until the weight of water displaced* is equal to the weight of the box. If an additional weight of a pound be put in the box, the box will sink until an additional pound of water is displaced. Now, a pound of water occupies 27·6 cubic inches, hence the additional volume displaced will be 27·6 cubic inches. If the box is 12″ long and 6″ broad, its horizontal surface is 72 square inches, and the box will sink $\frac{27 \cdot 6}{72}$ inches, *i.e.* ·38 inch.

NOTE.—In PROP. I., if the weight of the body be equal to the weight of the liquid displaced when totally immersed, it will float in any position, provided that the liquid is homogeneous. Thus a hollow brass sphere could be made such that its weight would be equal to the weight of the water it would displace; it would then float totally immersed in any position. If the body were solid and homogeneous, it would only float freely totally immersed in a liquid if its specific gravity were the same as that of the liquid.

The weight of a fish is about equal to the weight of water it displaces. In order to sink, some fishes are provided with an air-bag which they can exhaust, and so diminish their buoyancy; to rise, they fill the air vessel to *increase* their buoyancy.

Life-boats are also fitted with air-tight compartments to prevent their sinking.

If in PROP. II. the string which partly supports the body totally immersed were attached to the scale-pan of a balance, the pull of the string could be balanced by a weight equal to it in the other scale-pan. This weight is called the **apparent weight of the body in water**, and is therefore equal to the weight of the body less the weight of the fluid displaced by the body. Now, since the body is wholly immersed, the volume of fluid displaced is equal to the volume of the body. Therefore the weight of the fluid displaced by the body is equal to the weight of a volume of the fluid equal to the volume of the body.

If W = weight of body;
W' = apparent weight of body in the liquid;
w = weight of equal volume of liquid;
then $W' = W - w$; therefore $w = W - W'$.

The result that *the weight of equal volume of a liquid is equal to the difference of the real and apparent weights* is very important, as will be seen later.

It is sometimes said that a body loses a certain portion of its weight when weighed in water. For instance, if a block of iron loses $3\frac{1}{2}$ lbwt. in water, then $3\frac{1}{2}$ lbwt. is the weight of the water displaced by the iron.

The following examples are important, not only as examples of the preceding propositions, but on account of their application to the important problem of finding the specific gravity of a substance by experiment:

Ex. 1.—Five cubic feet of pine floats in water. How much is immersed? (Specific gravity of pine is ·5.)

Weight of five cubic feet of pine = weight of water displaced.

$$\therefore 5 \times \cdot 5 = V'.1.$$

$$\therefore V' = 5 \times \tfrac{1}{2} = 2\tfrac{1}{2} \text{ cubic feet.}$$

How much will be immersed in mercury? (Sp. gr. $= 13\frac{1}{2}$.)

$$5 \times \cdot 5 = V' \times 13\tfrac{1}{2};$$

$$\therefore 5 \times \tfrac{1}{2} = V' \times \tfrac{27}{2};$$

$$V' = \tfrac{5}{27} \text{ cubic foot.}$$

Ex. 2.—A cylinder of glass floats with its axis vertical in water, and it is observed that two-thirds of it is immersed. It is then placed in alcohol, and it is observed that five-sixths is now immersed. Compare the specific gravity of water and alcohol.

Let V = volume of the cylinder, then in the first case $\frac{2}{3}V$ is immersed; therefore weight of cylinder $= \frac{2}{3}V \times 1000$ ozwt.

In the second case, if S' = sp. gr. of alcohol,

$$\text{weight of cylinder} = \tfrac{5}{6}V \times S' \times 1000 \text{ ozwt.}$$

$$\therefore \tfrac{5}{6}.V.S'.1000 \text{ ozwt.} = \tfrac{2}{3}V.1000 \text{ ozwt.}$$

$$\therefore \tfrac{5}{6}.S' = \tfrac{2}{3};$$

$$S' = \tfrac{12}{15} = \cdot 8.$$

Ex. 3.—A cylinder of glass has a mark cut in it near one end. It is placed in water with the other end downwards, and a weight of 2 oz. is placed on it so as to make it sink until the mark is in the surface. It is then placed in alcohol, and is found to require a weight of two-fifths of an ounce to sink it to the mark; the weight of the cylinder being 6 ozwt., what is the specific gravity of alcohol?

Let V = volume of cylinder immersed in each case; S = sp. gr. ohol; then in the first case,

$$6 \text{ ozwt.} + 2 \text{ ozwt.} = V \times 1 \times 1000 \text{ ozwt.}$$

the second case,

$$6 \text{ ozwt.} + \tfrac{2}{5} \text{ ozwt.} = V \times S \times 1000 \text{ ozwt.}$$

$$\therefore 8 = V \times 1000;$$

$$6\tfrac{2}{5} = V \times S \times 1000.$$

$$\therefore \text{dividing } \frac{8}{6\frac{2}{5}} = \frac{1}{S}; \text{ or } S = \frac{6\frac{2}{5}}{8} = \tfrac{32}{40} = \cdot 8.$$

Ex. 4.—A piece of iron is weighed in air and found to be 25 lbwt.; it is then weighed in water and found to be $21\frac{2}{3}$ lbwt. Find the specific gravity of iron.

Weight of water displaced by iron $= 25 - 21\frac{2}{3} = 3\frac{1}{3}$ lbwt.

$$\text{Specific gravity of iron} = \frac{\text{weight of iron}}{\text{weight of equal volume of water}};$$

$$= \frac{25}{3\frac{1}{3}} = \frac{25 \times 3}{10} = 7{\cdot}5.$$

Ex. 5.—Show how to find the specific gravity of pine wood from the following results of experiment: Weight of the piece of wood is 5 lbwt.; weight of a piece of iron is 8 lbwt.; weight of iron in water, 6 lb. 15 ozwt. nearly; weight of both in water is 1 lb. 15 ozwt.

The object is to find the weight of water displaced by the wood.

(1) Weight of water displaced by iron and pine $= 13$ lb. $-$ 1 lb. 15 ozwt.
$= 11$ lb. 1 ozwt.

(2) Weight of water displaced by iron $= 8$ lb. $-$ 6 lb. 15 ozwt.
$= 1$ lb. 1 ozwt.

$\therefore$ Weight of water displaced by pine $= 10$ lbwt.

$$\text{But sp. gr. of pine} = \frac{\text{weight of pine}}{\text{weight of water displaced}};$$

$$= \tfrac{5}{10} = \tfrac{1}{2}.$$

NOTE.—If a body is at rest between two liquids which do not mix, so that part of the body is in one liquid and part in the other, then the resultant force of the fluid on the part in contact with the first liquid is equal to the weight of that liquid displaced; and the resultant force of the fluid on the part in contact with the second liquid is equal to the weight of the second liquid displaced. So that if V_1, V_2 are the volumes in the first and second liquids respectively, and S_1, S_2 are their specific gravities, then the resultant force of the fluid on the body is $(V_1S_1 + V_2S_2)\,1000$ ozwt.

If the body is floating *freely*, the weight of the body = resultant force of the fluid.

If the body is totally immersed, its volume is $V_1 + V_2$; and if its specific gravity $= S$, then its weight $= (V_1 + V_2)\,S.1000$ ozwt.

$$\therefore (V_1 + V_2)\,S = V_1S_1 + V_2S_2.$$

$$\therefore V_1(S - S_2) = V_2(S_1 - S);$$

$$\text{or } V_1 : V_2 :: S_1 - S : S - S_2.$$

Ex. 6.—A lump of metal of specific gravity 9·3 floats partly in oil of specific gravity ·9 and partly in mercury of specific gravity 13·5. What portion of its volume is in each?

Let V = volume in oil; V' = volume in mercury.

The weight of oil displaced $= V \times \cdot 9 \times 1000$ ozwt;

weight of mercury displaced $= V' \times 13{\cdot}5 \times 1000$ ozwt.;

and weight of metal $= (V + V')\,9{\cdot}3 \times 1000$ ozwt.

Therefore, since the metal floats freely,

$$(V + V')\,9{\cdot}3 = V \times \cdot 9 + V' \times 13{\cdot}5.$$

$$\therefore V(9{\cdot}3 - \cdot 9) = V'(13{\cdot}5 - 9{\cdot}3).$$

$$\therefore V\,8{\cdot}4 = V'\,4{\cdot}2.$$

$$\therefore V = \tfrac{1}{2}V'.$$

EXERCISES.

1. Find the resultant force of water on the following bodies totally immersed in it. Find also the weight of each body:

(a) A block of iron of 6 cubic feet volume (sp. gr. 7·5).
(b) A block of wood of 6 cubic feet volume (sp. gr. ·5).
(c) A cube of gold, 10 inches edge (sp. gr. 21).
(d) 205 cubic centimetres of cork (sp. gr. ·25).
(e) A prism of wood 8·5 cm. × 5·5 cm. × 4·35 cm. (sp. gr. ·5).

2. If the above bodies were totally immersed in mercury, find the resultant force of the liquid on each.

3. If the above bodies were immersed in alcohol (sp. gr. ·85), find the resultant force of the fluid on each.

4. Find the weight of each of the solids in Question 1, and then find whether the force which holds them totally immersed in water acts vertically downwards or vertically upwards. Find also the magnitude of the force in lbwt. or in gmwt.

5. Find the magnitude of the force which is required to hold the solids in Question 1 totally immersed in mercury, and say whether the force acts upwards or downwards.

6. Find the magnitude of the force which is required to hold the solids in Question 1 totally immersed in alcohol, and say whether the force acts upwards or downwards.

7. The volume of a piece of iron is 100 cc. What volume of mercury will have the same weight? If the iron floats in mercury, what volume will be below the surface?

8. A cube of wood 10 cm. edge floats in water. If it floats with three-fourths of its volume immersed, what is the weight of water displaced? What is the weight of the body? What additional force acting downwards upon it will hold it totally immersed?

9. What is the weight of 10 cc. of lead (sp. gr. 11·3)? What is the weight of 10 cc. of alcohol (sp. gr. ·85)? What is the apparent weight of 10 cc. of lead in alcohol?

10. The weight of a piece of iron is 30 lbwt. What weight of water will it displace? What weight of alcohol will it displace?

11. A solid weighs 205 gmwt. in air, and 178 gmwt. in water. What is the weight of water displaced? What is the volume of the solid in cc.? What is the density and specific gravity of the solid?

12. A solid (sp. gr. 7·5) weighs 205 gmwt. in air, and 185 gmwt. in alcohol. What is the weight of alcohol displaced? What is the volume of the solid? What is the weight of 1 cc. of alcohol?

Experimental Determination of Specific Gravities.

The following are the principal methods of finding the specific gravities of substances:

1. The **Specific Gravity Bottle.** This may be called the direct method.

(i) *To compare the specific gravities of two liquids.*

Take a thin glass bottle with a *hollow* perforated stopper—this is a specific gravity bottle for liquids.

(1) Weigh the bottle. Let the weight be W.

(2) Fill it with the first liquid and weigh it. Let the weight $= w$.

(3) Fill it with the second liquid and weigh it. Let the weight $= w'$.

Then weight of first liquid $= w - W$;

weight of second liquid $= w' - W$.

Therefore, since the volumes are the same,

sp. gr. of first liquid : sp. gr. of second liquid

$$:: w - W : w' - W.$$

(ii) *To find the specific gravity of a solid.*

Take a thin glass bottle with a perforated stopper (a specific gravity bottle for solids).

(1) Weigh the solid. Let the weight $= W$.

(2) Fill the bottle with water and weigh it. Let the weight $= w$.

(3) Now put the solid into the bottle, and let the weight $= w'$.

Now, $W+w=$ weight of solid, water, and bottle;

$w'=$ weight of solid, water, and bottle $-$ weight of water displaced by the solid.

$\therefore$ weight of water displaced by the solid $= W+w-w'$.

$$\therefore \text{sp. gr. of solid} = \frac{W}{W+w-w'}.$$

The advantages of the perforated stopper will be evident to the student who has found the volume of a solid by the displacement methods of Chapter II. The vessel there used has a sectional area of about 8 sq. cm., so that an error of 1 mm., *i.e.* ·1 cm., makes an error of ·8 cc. in the estimated volume. In using the specific gravity bottle, the bottle is filled with the liquid, and then the stopper is put on and thrust home, in so doing driving the liquid up the narrow tube of the stopper. In this way only a very small error can arise in filling the vessel. Before the empty bottle is weighed it must be carefully dried. This is done by gently heating it, and at the same time sucking the air out of it. To get rid of the air-bubbles which adhere to the solid, especially if it is in fragments, the water may be boiled with the solid in it; or the specific gravity bottle may be placed under the receiver of an air-pump and the air exhausted from it.

2. The **Hydrostatic Balance.** This method is an application of Prop. II., from which it has been shown that the weight of fluid displaced by a body $=$ weight of body in air $-$ weight of body in water.

If V cubic feet is the volume of the body, then the volume of the water displaced is V cubic feet.

$\therefore$ weight of water displaced $=$ weight of V cubic feet of water.

$$\therefore \text{sp. gr. of body} = \frac{\text{weight of body}}{\text{weight of volume } V \text{ of water}};$$

$$= \frac{\text{weight of body}}{\text{weight of water displaced}}.$$

3. The **Hydrometers.** These methods are applications of PROP. I., viz. that the weight of a body floating freely is equal to weight of liquid displaced.

The two hydrometers generally described are

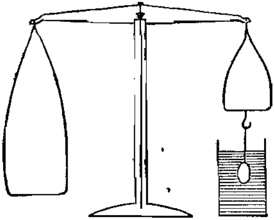
(i) The common hydrometer, which is used for finding the specific gravities of *liquids* only, and depends upon the fact that the weight floating in the liquid is constant. It may be called the *constant weight* hydrometer.

(ii) Nicholson's hydrometer, in which the volume immersed is kept constant. It may be called the *constant volume* hydrometer. It is used for *both liquids* and *solids*.

4. The **U tube method.** This method may be used to compare the specific gravities of two liquids which do not mix. The liquids are placed in the arms of the tube, and the heights of their surfaces above the common surface accurately measured. Then the specific gravities are inversely as these heights. (*See* Examples at end of Chap. III., and also experimental part at end of that chapter.)

The **Hydrostatic Balance** is an ordinary *sensible* balance, with one of the scale-pans suspended by shorter chains. This scale-pan has a hook below, from which to suspend the solid, which can therefore be easily weighed in a liquid as well as in air.

NOTE.—In a good sensible balance the beam turns about a sharp edge, which rests on two smooth hard supports. The scale-pans are suspended from sharp edges at the ends of the beam, and the



beam is provided with a long pointer to indicate the angle through which it turns. In using the hydrostatic balance the wire by which the solid is suspended should be very fine, and should be balanced by weights placed in the other scale-pan before the solid is attached. The water used should be first boiled to free it from the air which it holds in solution, and all air-bubbles which may adhere to the solid must be removed. It is advisable not to let the wire dip too much in the water, so that the error arising from the water which it displaces may be as small as possible.

(i) *To find the specific gravity of a solid of greater specific gravity than water.*

The solid whose specific gravity is required is suspended from the scale-pan, and weighed first in air and then in water.

Let W = weight in air;

W' = weight in water.

Then weight of water displaced $= W - W'$.

$$\therefore \text{sp. gr. of solid} = \frac{W}{W - W'}.$$

(ii) *To find the specific gravity of a solid of less specific gravity than water.*

In this case the solid will not sink in water. Take a heavier solid (called a sinker) which, joined to the lighter solid, will sink it in the water.

(1) Find the weight of the sinker in air, and then in water. The difference is the weight of the water displaced by the sinker.

(2) Find the weight of the sinker and solid together in air, and then in water. The difference is the weight of water displaced by sinker *and* solid.

(3) Find the weight of the solid. Let the weight $= W$.

The difference of (1) and (2) = weight of water displaced by the solid.

Let this weight $= W'$.

$$\therefore \text{sp. gr. of solid} = \frac{W}{W'}.$$

(iii) *To compare the specific gravities of two liquids.*

Take a solid which will sink in both liquids.

(1) Weigh the solid in the first liquid.

(2) Weigh the solid in the second liquid.

(3) Weigh the solid in air.

Difference of (1) and (3) is weight of first liquid displaced by solid. Let this weight $= W$.

Difference of (2) and (3) is weight of second liquid displaced by solid. Let this weight $= W'$.

Since the volumes of fluid displaced are the same in each case,

$\therefore$ sp. gr. of first fluid : sp. gr. of second fluid :: $W : W'$.

Ex.—A body weighs 250 grains in air, 40 grains in

water, and 50 grains in spirit. Find the specific gravity of solid and spirit.

Weight of water displaced by solid $= 250 - 40 = 210$ grs wt.

$\therefore$ sp. gr. of body $= \frac{250}{210} = \frac{25}{21}$.

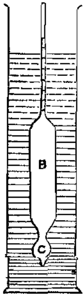
Weight of spirit displaced by solid $= 250 - 50 = 200$ grs wt.

Sp. gr. of spirit $= \frac{200}{210} = \frac{20}{21}$.

The **Common Hydrometer** is used to find the specific gravity of a liquid. It consists of a glass bulb B, with a thin stem at one end, and at the other end is a small bulb C containing a little mercury, so that the instrument may float vertically. The stem is graduated to give the volume of the hydrometer immersed when it floats in a liquid.

To compare the specific gravities of two liquids.

(1) Place the hydrometer in one liquid, and read the graduation at the point of the stem in the surface of the liquid.



This reading will give the volume immersed. Let the volume $= V$.

(2) Place the hydrometer in the second liquid and read the graduation. Let the volume immersed in this case $= V'$.

Now, weight of hydrometer = weight of first liquid displaced; also weight of hydrometer = weight of second liquid displaced.

$\therefore$ weight of first liquid displaced = weight of second liquid displaced.

Therefore, if S = specific gravity of first liquid, and S' = specific gravity of second liquid, then

$$V.S = V'.S';$$

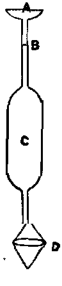
$$\therefore \frac{S}{S'} = \frac{V'}{V}.$$

NOTE.—*To graduate the common hydrometer.* It has been shown that if V, V' are the volumes of a common hydrometer immersed respectively in a liquid and in water, then the specific gravity of the liquid $= \frac{V'}{V}$. Now, instead of graduating the stem so as to give the volumes immersed, the instrument maker graduates it so as to give at once the specific gravity. This can be done in the following manner: Let U be volume of the hydrometer, let a be the area of the section of the stem, and suppose a length l' is above the surface when floating in water, and a length l when floating in a liquid, then

$$V' = U - al';$$
$$V = U - al.$$
$$\therefore \text{sp. gr. of liquid} = \frac{U - al'}{U - al}.$$

Thus, by taking a series of specific gravities in arithmetical progression, corresponding values of l can be found, since U, a, l' are supposed known.

Nicholson's Hydrometer consists of a hollow vessel C, with a long thin stem supporting a cup A attached to it. From the vessel is suspended a scale-pan D, which is weighted by means of a small bulb containing mercury. Near the top of the stem is a well-defined mark B, and in comparing specific gravities the instrument is made to sink in a fluid to this mark by means of weights placed in A.



(i) *To find the specific gravity of a solid.*

(1) Place the hydrometer in water, and put weights in A until B is in the surface of the water.

(2) Now place the solid in A, and take out weights until B is again in the surface.

(3) Now place the solid in D, and *add* weights until B is again in the surface.

The weight taken out in the second case = weight of solid. Let this weight = W.

The weight added in the third case = weight of water displaced by the solid. Let this weight = W'.

$$\therefore \text{ sp. gr. of the solid} = \frac{W}{W'}.$$

If the weight in A in first case $= w$;
„ A in second case $= w'$;
„ A in third case $= w''$;

then $W = w - w', \quad W' = w'' - w'$.

$$\therefore \text{ sp. gr.} = \frac{w - w'}{w'' - w'}.$$

(ii) *To find the specific gravity of a liquid.*

(1) Weigh the hydrometer. Let its weight be W.

(2) Place the hydrometer in the liquid, and put weights in A until B is in the surface. Let the weights = w.

(3) Place the hydrometer in water, and put weights in A until B is in the surface. Let the weights = w'.

Then weight of liquid displaced = $W + w$;
weight of water displaced = $W + w'$.

And since the volume displaced is the same in each case,

$$\text{sp. gr. of the liquid} = \frac{W + w}{W + w'}.$$

Ex.—It is found that 8 ozwt. will sink a Nicholson's hydrometer to the mark. A solid is put in the upper cup, and the weight required to sink the instrument is 2 o[illegible]

The solid is now put in the lower cup, and the weight required is $2\frac{1}{2}$ ozwt. Find the specific gravity of the solid.

$$\text{Weight of solid} = 8 \text{ ozwt.} - 2 \text{ ozwt.} = 6 \text{ ozwt.}$$

$$\text{Weight of water displaced by solid} = 2\tfrac{1}{2} \text{ ozwt.} - 2 \text{ ozwt.} = \tfrac{1}{2} \text{ ozwt.}$$

$$\therefore \text{sp. gr. of solid} = \frac{6}{\frac{1}{2}} = 12.$$

NOTE.—In using Nicholson's hydrometer, it will be found that with given weights in the cup the instrument will rest in any position within a small range. To avoid this error, put weights into the cup so that the mark is just above the surface, then put weights until it is just below the surface. The mean of the weights in the cup in each case will be very approximately the true weight.

It has been assumed throughout that the weight of a given volume of water is the same at all temperatures. This is nearly, but not quite, true. The specific gravity of water decreases with the rise of temperature from 4° C. It is therefore necessary in accurate experiments to observe the temperature of the water used, and find from the physical tables the specific gravity of water at that temperature.

If $\sigma =$ sp. gr. of water used;
$S =$ sp. gr. of the substance;
$V =$ volume of the substance;

then $V \,.\, S \,.\, 1000$ ozwt. $=$ weight of the body;
$V \,.\, \sigma \,.\, 1000$ ozwt. $=$ weight of the water displaced.

$$\therefore \frac{S}{\sigma} = \frac{\text{weight of the body}}{\text{weight of the water displaced}}.$$

The specific gravity found by the methods described must therefore be multiplied by the specific gravity of water at the observed temperature. The specific gravity of water at 15° C. (the mean temperature in England) is ·99915, so that the correction is very small.

It has also been assumed that the weight of a body in air is the same as in vacuum. It will be proved in the next chapter that air has weight, and therefore a body really weighs less in air, by the weight of air displaced by the body. Suppose the volume of a body is 1000 cc. and the specific gravity of air ·0013, then the weight of

air displaced is 1·3 gmwt. Hence if the body is iron, and weighs 7500 gmwt. in air, its true weight is 7500 − 1·3, *i.e.* 7498·7 gmwt. The difference is comparatively small. If the body is cork, and weighs 200 gmwt. in air, its true weight is 198·7 gmwt., which is more appreciable. In finding therefore the true weight of a body, a correction must be made for the weight of air displaced both by the body itself and by the standard weights. Now, it is known that the weight of the air will depend upon the density of the air, and this depends upon the temperature, pressure, and condition of the atmosphere as regards the moisture it contains. Methods of determining the density of the air in a room at a given time are described in the subject of Heat.

[*The remainder of this Chapter may be omitted in a first course of reading.*]

PROP. IV.—*Conditions of equilibrium of a body immersed in a liquid and acted upon by any number of forces.*

In this case the body is acted upon by the given forces, by its own weight, and by the force of the fluid. This last is equal to the weight of fluid displaced, and acts through the centre of gravity (C.G.) of the fluid displaced. Hence the conditions of equilibrium can be found by ordinary statics.

An elementary example will be given. A body

immersed in a fluid is capable of turning without friction about a fixed point C, and is acted upon by a force at D vertically downwards.

Let W lbwt. be the weight of the body, and C.G. G.

Let W' lbwt. be the weight of water displaced, and C.G. of fluid displaced H.

Let P lbwt. be the force acting at D.

Let the directions of these forces meet the horizontal line through C in G, A, D respectively.

Then, taking moment about C,

$$W'.AC = W.GC + P.CD.$$

Ex.—The centre of the ball of a ball-tap is 2 feet from the axis. The piston, 1 inch diameter, which closes the inlet-pipe, is 2 inches from the axis. The diameter of the ball is 8 inches, and the pipe is closed when the ball is half immersed. Find the pressure of the water in the pipe when the C.G. of the ball-tap is 1 inch from the axis, and weight of ball-tap is just balanced by a counter-balance weight.

Here volume of water displaced $= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times 4^3$ cubic inches.

And weight of water displaced $= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times 4^3 \times \frac{1000}{1782 \times 16}$ lbwt.

$= 4{\cdot}8$ lbwt. nearly.

Hence $\qquad 4{\cdot}8 \times 24 = P.2;$

$P = 57{\cdot}6$ lbwt.

Now, the area of the section of the pipe $= \frac{22}{7} \times \frac{1}{4}$ sq. inch;

$= \frac{11}{14}$ sq. inch.

$\therefore$ force of the water on $\frac{11}{14}$ sq. inch is $57{\cdot}6$ lbwt.

$\therefore$ pressure of the water $= 57{\cdot}6 \div \frac{11}{14}$, *i.e.* $\frac{57{\cdot}6 \times 14}{11}$ lbwt. per sq. in.

$= 73{\cdot}3$ lbwt. per sq. inch.

On the stability of bodies floating in water.

If a body floats in water, it is, as we have seen, acted upon by two parallel, equal, and opposite forces, viz. the weight of the body acting vertically downwards through the C.G. of the body, and the thrust of the liquid acting

vertically upwards through the centre of gravity of the liquid displaced. When the body is in a position of rest, the directions of these two forces are coincident, and the two centres of gravity are in the same vertical. For any given body there may be several positions in which this condition is satisfied, *i.e.* there may be several positions of rest. For example, a rectangular prism of uniform density will rest with any face horizontal and immersed; a cylinder of uniform density will rest with its ends horizontal, and it will also rest in any position with its axis horizontal; a sphere of uniform density will rest in any position whatsoever. But anyone who has tried to make an oblong piece of wood float with its longer sides vertical, or to make a cylinder like a lead-pencil float vertically, is at once struck with the difference between the various positions of equilibrium. If a body be slightly disturbed from a position of rest, it may either fall back to this position, or fall away to another position of rest, or it may remain at rest. In the first case the body is said to be in a position of "stable" equilibrium, in the second of "unstable" equilibrium, in the third of "neutral" or "indifferent" equilibrium. A rectangular prism floating with its shorter edges vertical is (when the shorter edge is not more than about five-sevenths of either of the others) in a position of stable equilibrium, and, if slightly disturbed, it will return to its position of rest. On making the horizontal face broader, it will be found that the prism will return to its position of rest much more quickly, the forces required to disturb it are greater, and it is therefore in a position of greater stability. The consideration of the positions of stable equilibrium of ships, of the nature of the stability, and of the magnitude of the righting couple as the body is turned away from its position of rest, is of the greatest importance in the construction of ships, and forms one of the essential operations of a naval architect, and the following

explanation will introduce the student to the main principles of the subject.

The body will be supposed a prism, and the centres of gravity and of buoyancy will be in the same vertical plane when the axis of the prism is horizontal. If the body is slightly disturbed from its position of rest by rotating through a small angle about a line parallel to the axis, the new centre of buoyancy will not generally be in the same vertical with the centre of gravity. Suppose Fig. 1 represents a section of the body through C.G. perpendicular to the axis. Let G be C.G. and H centre

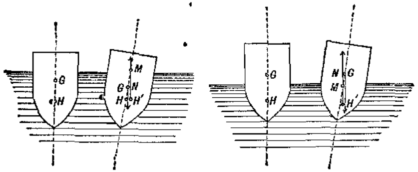


Fig. 1. Fig. 2. Fig. 3. Fig. 4.

of buoyancy. When in position of rest GH is vertical. Now, suppose the body is turned through a small angle about G so as to occupy position in Fig. 2. Then the new centre of buoyancy will be at H'. If the body is left free to move, it will be acted upon by its weight vertically downwards through G, and the force of the fluid vertically upwards through H', and these will be equal in all positions. These two equal forces will form a couple. Draw the vertical through H' to meet HG in M; then if, as in Fig. 2, the point M is above G, the couple is *such as to tend to turn the body back to its* original position. If, as in Fig. 4, the point M is below G, the couple will tend to turn the body *further away* from its original position. The position of M for very

slight displacements is practically the same point, and its distance from H can be found either by graphic methods or by analytic methods. Provided the volume displaced is the same, the distance HM is independent of the position of G in the line HG. This point M is called the **metacentre** of the body in this position of rest and for small rotation about a line parallel to the axis. Hence the body is in stable equilibrium if the C.G. is below the metacentre, and in unstable equilibrium if it is above the metacentre. It may happen that the new position of the centre of buoyancy is in the same vertical with C.G., *i.e.* that the metacentre and C.G. coincide. In this case the couple vanishes, and the body remains at rest in its new position, this body being in neutral equilibrium. An example of neutral equilibrium is a cylinder with C.G. in its axis. The centre of buoyancy is easily seen in both cases (Fig. 5 and Fig. 6) to be vertically below G. The metacentre is at G.

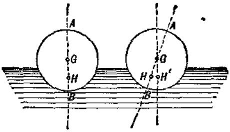
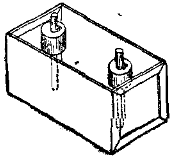


Fig. 5. Fig. 6.

The *degree of stability* will depend upon the magnitude of the righting couple, and this will depend upon the weight of the body and upon the arm of the couple. The weight of a ship is generally expressed in tons-weight, and the arm or perpendicular distance GN (Fig. 2) is expressed in feet, therefore the magnitude of the righting couple is expressed in foot-tons-weight. To increase the stability the height GM (called the metacentric height)

should be made as large as convenient. This can be done by keeping the centre of gravity as low as possible, and by raising the centre of buoyancy, thereby raising the metacentre. It can be shown by analysis that the length *HM* varies as the cube of the breadth of the surface of floatation, and inversely as the volume immersed. Hence a small increase in breadth considerably increases the stability. This can be shown by taking three rectangular prisms of pine wood, each 4 inches long and 1 inch deep, and respectively 1, 2, and 3 inches wide. It will be found that the 3-inch wide prism will float in stable equilibrium with its 4-by-3-inch face horizontal; so will the 2-inch prism, but in much less stable condition, as can be seen by the slowness of returning to rest, whereas the first prism will not float with the 4-by-1-inch face horizontal, but falls away to the position in which a diagonal of the small end is horizontal. Interesting experiments in illustration of "stability" can be performed with a box $8'' \times 4'' \times 4''$, fitted with two weights equally distant from the ends, so arranged as to be capable of displacement. The figure will explain the construction. By gradually raising the weights the point is reached when the box begins to fall away from the vertical. The line through the position of the weights will then pass through the metacentre.



EXPERIMENTS ON CHAPTER IV.

The experiments in this chapter are divided into two distinct parts. The first part consists of a series of experiments to prove the truth of the principle of Archimedes. The second part gives the method of finding the specific gravities, volumes, and densities of bodies, assuming the truth of this principle.

PART I.

TO PROVE THE PRINCIPLE OF ARCHIMEDES BY EXPERIMENT.

The principle of Archimedes states that the upward vertical thrust (or buoyancy) of a liquid on a solid immersed in it is equal to the weight of the fluid displaced by the solid. In order to prove it, two quantities must be measured independently: (1) the resultant upward fluid thrust; (2) the weight of liquid displaced. If a body is suspended by a string, the pull on the string will be equal to the weight of the body. And if the string is attached to the end of one arm of a balance, this pull can be measured by weights in the other scale-pan. If now the body be totally immersed in a liquid, the pull on the string will be less (the body appears to have lost weight), and this difference in the pull is due to the upward thrust, and is equal to it. The upward thrust is therefore equal to the difference between the weight in air and the weight in water. If the body floats in water, the upward thrust upon it when it floats freely will just support the weight of the body, and in this case the upward thrust will be equal to the weight

EXP. I.—*To prove the principle of Archimedes when the solid is totally immersed.*

Take the solid iron or brass cylinder used in Exp. I. Chap. I.

1st. *To find the upward thrust when totally immersed in water.*

Suspend the solid by a fine cotton thread from the counterpoise. (*See* Note *opposite.*)

(1) Weigh it in air.

(2) Now bring a vessel containing water, and place it under the solid so that the solid dips into it. Put weights into the scale-pan until there is equilibrium, with the solid totally immersed. This will give the weight of the solid in water.

The difference between the weights in (1) and (2) will be equal to the upward thrust.

2nd. *To find the weight of water displaced by the solid when totally immersed.*

Take a hollow brass vessel in which the iron solid will just fit, so that the volume of water the vessel will hold is equal to the volume of the solid, *i.e.* is equal to the volume of water the solid will displace when it is totally immersed.

(1) Weigh the vessel empty.

(2) Fill it carefully to the brim, and weigh it full of water.

The difference between (1) and (2) is equal to the weight of water the solid will displace.

It will be found that the upward thrust = weight of water displaced.

Example.

(1) Weight of solid in air 206·5 gmwt.
(2) Weight of solid in water 179·65 gmwt.
(3) Weight of brass vessel empty 37·25 gmwt.
(4) Weight of brass vessel full of water 64·75 gmwt.

Calculation.

From (1) and (2):

Upward thrust of water on the solid = 206·5 − 179·65 gmwt.
= 26·85 gmwt.

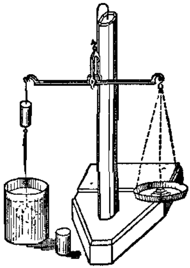
From (3) and (4):

Weight of water in vessel, *i.e.* weight of water displaced by the solid = 64·75 − 37·25 gmwt.
= 27·5 gmwt.

The upward thrust differs from the weight of water displaced by 27·5 − 26·85 gmwt., *i.e.* by 0·65 gmwt.

The somewhat large difference here is due to the difficulty of measuring the weight of water which the vessel contains, it being difficult to tell with accuracy when the vessel is filled.

NOTE.—In order to weigh a solid in a liquid, one of the scale-pans of the balance must be taken off and replaced by a counterpoise. Such a counterpoise can easily be made from a piece of lead, filed down to the right weight, and fitted with a hook at one end and an eye at the other. Care should be taken to see that the solid



is totally immersed when there is equilibrium, and that no bubbles of air adhere to it. It is better to use distilled water which has been boiled and allowed to cool. If the liquid is a viscous liquid, like glycerine, time must be given for the body to take up its position of

EXP. I.*a*.—The weight of the water displaced by the solid can be found by first finding the volume of the solid by direct measurement, and hence finding the weight of an equal volume of water.

Thus the volume of the solid used in Exp. I. was found (*see* Exp. I. Chap. II.) to be 26·95 cc. Therefore the weight of water displaced = 26·95 gmwt., and the difference between the upward thrust and the weight of water displaced is only ·1 gmwt.

EXP. II.—*To prove the principle of Archimedes for any liquid.*

This is done as in Exp. I., using the given liquid instead of water.

Example. Liquid: Methylated spirits.

(1) Weight of cylinder in air 206·5 gmwt.
(2) Weight of cylinder in the liquid 183·7 gmwt.
(3) Weight of brass vessel 37·3 gmwt.
(4) Weight of brass vessel full of the liquid 60·45 gmwt.

From (1) and (2) upward thrust = 22·8 gmwt.
From (3) and (4) liquid displaced = 23·15 gmwt.

EXP. III.—*To prove the principle of Archimedes for any irregular-shaped solid which sinks in water.*

(1) Find the upward thrust as in Exp. I. 1st.

(2) Find the weight of water displaced as in Exp. III. ii. Chap. II.

Example. Solid: A piece of zinc.

(1) Weight of solid in air 55·2 gmwt.
(2) " " water 47·25 gmwt.
Upward thrust = 7·95 gmwt.

(3) Weight of vessel, water, and zinc outside 242·65 gmwt.
(4) " " " " inside 234·65 gmwt.
Weight of water displaced = 8·00 gmwt.

This gives only a difference of ·05 gmwt.

Exp. IV.—*To prove the principle of Archimedes for a solid totally immersed in a liquid, the solid having a less specific gravity than the liquid.*

Suppose the solid is a piece of wood which will float in water. Take a solid sinker.

(1) Find the upward thrust on the sinker when totally immersed (Exp. I. 1st).

(2) Tie the solid and sinker together, and find the upward thrust on both when totally immersed (Exp. I. 1st).

(3) Find the weight of water displaced by the sinker. (Exp. III. ii. Chap. I.)

(4) Find the weight of water displaced by the solid and sinker. (Exp. III. ii. Chap. I.)

From (1) and (2) the upward thrust upon the solid alone can be found.

From (3) and (4) the weight of water displaced by the solid alone can be found.

Example. Solid: A piece of pine wood.

(1) Weight of sinker in air 51·2 gmwt.

(2) Weight of sinker in water 43·7 gmwt.

∴ upward thrust on the sinker = 7·5 gmwt.

(3) Weight of solid and sinker in air 63·7 gmwt.

(4) Weight of solid and sinker in water 34·8 gmwt.

∴ upward thrust on the solid and sinker = 28·9 gmwt.

∴ the upward thrust on the solid = 28·9 — 7·5 gmwt. = 21·4 gm

(5) Weight of vessel, water, and sinker 194·3 gmwt.

(6) Weight of vessel, water, and sinker inside 187 gmwt.

∴ weight of water displaced by the sinker = 7·3 gmwt.

(7) Weight of vessel, water, and solid and sinker 206·8 gmw

(8) Weight of vessel, water, with solid and sinker inside 178·8 gm

∴ weight of water displaced by the solid and sinker = 28·0 gmw

∴ weight of water displaced by the solid alone = 28·0 — 7·3 gm = 20·7 gmwt.

This shows a difference of ·7 gmwt.

EXP. V.—*To find the upward thrust of a liquid on a solid which floats in it.* (*See* PROP. I.)

(i) Take a rectangular block of wood 5 cm. square and 4 cm. deep. Coat it with varnish to prevent water soaking into it.

Now let it float in water with its short edges vertical, and measure the length of the short edge which is above the surface. Hence find the length of the edge below the surface of the water. Suppose this length is 1·8 cm. Then, since the area of the horizontal section is 25 sq. cm., the volume immersed is 25 × 1·8 cc., *i.e.* 45 cc.; and therefore the weight of water displaced is 45 gmwt.

Now find the weight of the block.

The upward thrust of the water upon the block must be equal to the weight of the block (PROP. I.).

It will be found that this upward thrust is equal to the weight of water displaced by the block.

(ii) Now place a 10 gm. weight on the centre of the top face, so that the block still floats with its short edges vertical. Find the length of the edge below the surface, and prove that the weight of water now displaced is equal to the weight of the block + 10 gmwt., *i.e.* equal to the weight of the floating body.

(iii) Find what weight will just cause the block to float totally immersed.

It is difficult to get a piece of wood to float with its face quite horizontal. If the face is not quite horizontal, make four paper *millimetre scales, and attach one to each of the four vertical edges,* and read the depth of each edge below the surface. The mean of these depths may be taken as the true depth if the wood floated horizontal. *On adding weights at the centre of the top face, it will* be found that the block falls more and more away from the position in which the edges are vertical. It will be found necessary to attach the weights to the bottom of the wood, and correct for the water which they displace.

PART II.

TO FIND THE SPECIFIC GRAVITY, VOLUME, AND DENSITY OF BODIES, ASSUMING THE PRINCIPLE OF ARCHIMEDES.

In all the following experiments it is necessary to find the weight of a liquid displaced by a solid. In Chap. II. this weight has been found without assuming any hydrostatic principle, but in this chapter the weight of liquid displaced by a solid will be found from the fact proved in Prop. II. that the weight of liquid displaced by a solid is equal to the difference between the weight of the solid in air and in the liquid.

EXP. VI.—*To find the density of a liquid*, e.g. *water.*

The principle of the method is the same as that in Exp. III. Chap. II., but the weight (and therefore the mass) of the liquid displaced by the solid is found from the fact that it is equal to the difference between the weight of the solid in air and its weight in water.

Take a solid of a regular shape, the volume of which can be found by direct measurement.

(i) Find its volume. (Exp. I. Chap. II.)

(ii) To find the weight of water it will displace.

(1) Weigh the solid in air.

(2) Weigh the solid totally immersed in water.

The difference between (1) and (2) is equal to the weight of water displaced.

Example.

(i) The volume of the iron cylinder was found, as in Exp. I. Chap. II., to be 26·95 cc.

(ii) (1) Weight of the solid in air 206·5 gmwt.

(2) Weight of the solid in water 179·65 gmwt.

Calculation.

From (ii) weight of water displaced by the solid $= 26{\cdot}85$ gmwt.

∴ the mass of water displaced by the solid $= 26{\cdot}85$ gm. and the volume of the solid, and therefore the volume of this water $= 26{\cdot}95$ cc.

$$\therefore \text{the mass of } 26{\cdot}95 \text{ cc. of water} = 26{\cdot}85 \text{ gm.}$$

$$\text{The mass of 1 cc. of water} = \frac{26{\cdot}85}{26{\cdot}95} \text{ gm.}$$

$$= {\cdot}9963 \text{ gm.}$$

∴ the density of water is found to be ·9963 gm. per cc.

In a similar manner the density of any other liquid can be determined.

EXP. VII.—*To find the specific gravity and volume of a solid which will sink in water.* (*See* i. p. 91.)

Example. Solid: Iron.

Weight in air 66·35 gmwt.

Weight in water 57·71 gmwt.

∴ weight of water displaced $= 8{\cdot}64$ gmwt.

$$\therefore \text{sp. gr. of iron } \frac{66{\cdot}35}{8{\cdot}64} = 7{\cdot}679.$$

Find in the same way the sp. gr. of brass, copper, zinc, sulphur, and glass.

The weight of water displaced by the solid $= 8{\cdot}64$ gmwt., and 1 gmwt. of water occupies a volume of 1 cc.

∴ the volume of the water displaced $= 8{\cdot}64$ cc.

∴ the volume of the solid $= 8{\cdot}64$ cc.

EXP. VIII.—*To find the specific gravity of a solid which will float in water.* (*See* ii. p. 92.)

[Ex]ample. Solid: Paraffin.

(1) Weight of paraffin in air 13·65 gmwt.
(2) Weight of sinker in air 55·25 gmwt.
(3) Weight of sinker in water 47·25 gmwt.
(4) Weight of sinker and paraffin in water 45·05 gmwt.

[Cal]culation.

From (2) and (3) weight of water displaced by the sinker
$= 55{\cdot}25 - 47{\cdot}25$ gmwt.;
$= 8{\cdot}0$ gmwt.

From (1) (2) and (4) weight of water displaced by solid and sinker $= 13{\cdot}65 + 55{\cdot}25 - 45{\cdot}05$ gmwt.;

$\therefore$ weight of water displaced by paraffin alone $= 23{\cdot}85 - 8{\cdot}0$ gmwt.
$= 15{\cdot}85$ gmwt.

$$\therefore \text{sp. gr. of paraffin} = \frac{13{\cdot}65}{15{\cdot}85} = {\cdot}861.$$

Find the volume of the solid.
Find the sp. gr. of deal, mahogany, cork, and wax.

Exp. IX.—*To find the specific gravity of a so[lid] which will dissolve in water.*

Take a liquid which will not dissolve the solid. Find its specific gravity. (Exp. VI. Chap. II. *or* Exp. X. *below.*)

Find the weight of liquid displaced by the solid. (Exp. VII.)

Hence find the weight of equal volume of water. (*See* Ex. 5, p. 22.)

Hence find the specific gravity of the solid.

Example.

Weight of solid 55·25 gmwt.
Weight of solid in turpentine 48·45 gmwt.
Sp. gr. of turpentine ·86.

Calculation.

Weight of turpentine displaced by the solid $= 6{\cdot}8$ gmwt.

$$\therefore \text{weight of equal volume of water} = \frac{6{\cdot}8}{{\cdot}86} = 7{\cdot}907 \text{ gmwt.}$$

$$\therefore \text{sp. gr. of solid} = \frac{55{\cdot}25}{7{\cdot}907} = 6{\cdot}985.$$

Exp. X.—*To find the specific gravity of a liquid.*

Take a solid which will sink both in the liquid and in water, and find the weight of each displaced by the solid. (*See* iii. p. 92.)

Example. Liquid: Glycerine.

Weight of steel cylinder in air 206·55 gmwt.
Weight of steel cylinder in water 179·65 gmwt.
Weight of steel cylinder in glycerine 172·95 gmwt.

$$\text{Hence sp. gr. of glycerine} = \frac{33{\cdot}6}{[illegible]} = 1{\cdot}249.$$

EXP. XI.—*To find the sectional area and thickness of a wire.*

Take as long a piece of the wire as convenient.

(1) Find its length.

(2) Find its weight in air.

(3) Find its weight in water.

Hence find its volume; and hence deduce sectional area and diameter.

Example.

Length of wire (mean of 5 measurements) 122·64 cm.

Weight in air („ „) 12·161 gmwt.

Weight in water, corrected for weight of cotton (mean of 5 measurements) 10·755 gmwt.

Calculation.

Weight of water displaced by the wire $= 12{\cdot}161 - 10{\cdot}755$ gmwt.
$= 1{\cdot}406$ gmwt.

$\therefore$ volume of the wire $= 1{\cdot}406$ cc.

$$\therefore \text{ sectional area of the wire} = \frac{1{\cdot}406}{122{\cdot}64} \text{ sq. cm.} = {\cdot}01146 \text{ sq. cm.}$$

If r cm. is the radius of the sectional, then area of the section is $3{\cdot}1416\, r^2$ sq. cm.

$$\therefore 3{\cdot}1416\, r^2 = {\cdot}01146.$$

$$\therefore r^2 = \frac{{\cdot}01146}{3{\cdot}1416} = {\cdot}003647 \text{ sq. cm.} = {\cdot}3647 \text{ sq. mm.}$$

$$\therefore r = {\cdot}604 \text{ mm.}$$

And the thickness of the wire $= 1{\cdot}208$ mm.

EXPERIMENTAL EXERCISES.

1. Make a solution of alcohol and water so that bees-wax will just float in it totally immersed. Find the sp. gr. of the wax from the sp. gr. of the solution.

2. Find the sp. gr. of two liquids which do not act chemically upon each other, and which mix. Mix them in the ratio of 2 vols to 1 vol, and find the sp. gr. of the mixture. Test your result by theory.

3. A solid is given you of sp. gr. 7·5. Find its volume.

4. Pour water into a small vessel, and weigh the vessel. Suspend a solid from a fixed support so that it is totally immersed in the water, and find the weight required to balance the vessel. Prove that the difference in the weights is equal to the weight of water displaced by the solid.

EXAMPLES ON CHAP. IV.

1. Find the upward force of the liquid on the following bodies:

(a) 50 cc. of iron half immersed in water.
(b) 25 cubic feet of wood half immersed in water.
(c) 60 cubic feet of copper two-thirds immersed in mercury.
(d) 67 cubic inches of silver totally immersed in water.
(e) 500 cc. of lead totally immersed in alcohol.

2. Find the weight of each of the bodies in Question 1, and say whether they will float in the liquid or not.

3. Find the weight of 100 cc. of cork (sp. gr. ·25), and find the volume of water which will have the same weight. Hence find what volume of the cork will be above surface if it float in water.

4. Find the weight of 27 cc. of iron, and find the volume of mercury which will have the same weight. Hence find the volume of iron which will be immersed if it float in mercury.

5. Find the weight of water displaced by 27 gm. of iron. What is the weight of this iron in water? Find the weight of glycerine displaced by the iron if totally immersed in it, and hence find its weight in glycerine.

6. Find the weight in alcohol of 60 lb. of lead.

7. A body is attached to a spring balance, one end of which is fixed to the bottom of a vessel containing water.

The spring is found to be stretched to the point indicating 75 gmwt. when the body is totally immersed. The body itself weighs 25 gmwt. Find its sp. gr.

8. Find the weight of water displaced by 100 lb. of copper. Find the weight of glycerine displaced by it. Hence find the weight of the copper in water and in glycerine.

9. What weight of iron must be taken so that it will keep 100 cc. of cork totally immersed in water?

10. What force must be exerted upon a cube which weighs 850 gmwt., and has a volume of 1000 cc. in order to keep it totally immersed in water? What force must be exerted upon it to keep it totally immersed in alcohol? (Sp. gr. ·85)

11. What force must be exerted to suspend 150 cc. of iron totally immersed in water? What volume of platinum would be suspended totally immersed in water by the same force? Give an easy method of testing the result experimentally.

12. A hollow cube 10 inches edge floats half immersed in water. What force is necessary to increase its depth by 1 inch? What weight of lead placed inside the cube would increase its depth by 2 inches? What additional weight of lead would be required to sink it 2 inches if the lead were attached to the bottom of the cube, but outside it?

13. The area of the "water-line" of a ship is 3000 square feet. What depth will the ship sink if 100 tons be placed in it?

14. A rectangular prism of wood 12 inches long, 5 inches broad, and 2 inches deep floats in water. What weight will be required just to cause it to float totally immersed? (Sp. gr. of the wood ·5.)

15. A block of wood of specific gravity ·75 weighs 500 gmwt. What volume of water will it displace if it is floating freely in it?

16. What is the weight in water of 60 gm. of copper tied to 10 gm. of cork? What will their weight in turpentine be?

17. A cylindrical pencil is arranged so as to float vertically in water and in another liquid. It floats with half its length immersed in water, but with only one-third its length in the liquid. Find the specific gravity of the liquid.

18. A vessel containing water is placed in one scale-pan of a balance, and balanced by weights in the other scale-pan. A piece of lead weighing 20 gmwt. is suspended by a string from a fixed support, and is placed totally immersed in the water. What additional weight must be used to restore the balance?

19. A body floats with one-tenth of its volume above the surface of pure water. What fraction of its volume would project above the surface if it were floating in liquid of sp. gr. 1·25?

20. An iron cylinder 5 cm. long and 5 cm. diameter floats in mercury with its axis vertical. What length of its axis is below the surface?

21. An iron cylinder is 20 cm. long and 5·5 cm. diameter. Find its weight in water.

22. What force will be required to keep the cylinder in Question 21 totally immersed in mercury?

23. A cube of 15 inches edge is made of copper. Find its weight in air and in water.

24. Find the weight of a glass marble (1·5 cm. in diameter) in air and in water.

25. A piece of iron weighing 200 gmwt. floats in mercury of sp. gr. 13·6 with five-ninths of its volume immersed. Find the volume and specific gravity of the iron.

26. A solid floats in water with two-thirds of its volume above the surface. If it is placed in mercury, what portion of its volume will

27. A solid floats half immersed in water. What weight of iron will cause it to float with three-fourths of its volume below the surface (1) when the iron is above the surface; (2) when it is below the surface? (Sp. gr. of iron = 7·5.)

28. Find the apparent weight of 5 cubic inches of gold (sp. gr. = 19·3) in mercury (sp. gr. = 13·5).

29. What will be the force required to keep a block of 27 cubic feet of pine wood (sp. gr. = ·5) below the surface of water?

30. What will be the tension (or pull) of a string which supports a block of 27 cubic feet of pine wood with two-thirds of its volume above the surface of water?

31. Five cubic inches of iron (sp. gr. = 7·5) float in mercury. How many cubic inches are above the surface?

32. It is observed that a cylinder of glass (sp. gr. = 3·5) floats vertically with $\frac{20}{27}$ of its length above the surface of a liquid. What is the specific gravity of the liquid?

33. A man weighing 160 lb. floats with 4 cubic inches of his body above the surface. What is his volume in cubic feet?

34. A cylindrical bucket 10 inches in diameter and 1 foot high is half full of water. Half-a-hundredweight of iron is suspended in it, and then placed on the bottom. How much will the force of the water on the bottom be increased in each case?

35. A piece of wood weighing 120 lb. floats in water with four-fifths of its volume in the water. Find the volume of the wood.

36. A cube of wood floating in water supports a weight of 480 oz. On the weight being removed it rises 1 inch. Find the size of the cube.

37. A piece of copper weighs 10 lbwt. in air and $8\frac{3}{4}$ lbwt. in water. Find its specific gravity and its volume in cubic inches.

38. A lump of marble weighs 5 lbwt. in air and $4\frac{1}{6}$ lbwt. in water. Find its specific gravity.

39. A cube, the edge of which is 4 inches long, weighs 16244 grswt. in air and 95 in water. Find the weight of a cubic inch of water, the specific gravity of air being $\frac{1}{770}$.

40. Two bodies, whose specific gravities are ·24 and ·6 respectively, balance each other under water. Find the ratio of their weights.

41. A piece of metal of specific gravity 9·8 weighs in water 56 grswt. What is its true weight?

42. A solid, the volume of which is 1·6 cubic inches, weighs 3·4 grswt. in a fluid of specific gravity ·85. Find the specific gravity and weight of the solid.

43. A body weighing 300 grammes, and of specific gravity 5, has 100 grammes of another substance attached to it, and the joint weight of the two in water is 300 gmwt. Find the specific gravity of the attached weight.

44. It is required to find the specific gravity of potassium, which decomposes water. A lump weighing 432·5 grswt. in air is weighed in naphtha, the specific gravity of which is ·847, and is found to weigh 9 grswt. What is the specific gravity?

45. *A ball of glass weighing 665·8 gmwt. in air is found* to weigh 465·8 gmwt. in water and 297·6 gmwt. in sulphuric acid. What is the specific gravity of the latter?

46. A piece of cork whose weight is 19 ozwt. is attached to a bar of silver weighing 63 oz., and the two together just float in water. The specific gravity of silver is 10·5 times that of water. Find the specific gravity of the cork.

47. A bar of metal weighs 1275 gmwt. in air, 1147·5 gmwt. in spirit, and 1125 gmwt. in water. Find the specific gravities of the metal and of the spirit.

48. A piece of copper weighs in air 137·28 gmwt., and in water 121·68 gmwt. Show that the density (ratio) of

49. A piece of copper when weighed in air is balanced by 125·35 gm. What will it weigh in water if the density (ratio) of copper be 8·8?

50. A substance which weighs 14 lbwt. in air and 12 lbwt. in water floats in mercury whose specific gravity is 13·6. *What proportion of its volume will be immersed?*

51. Two bodies, one of which weighs 7 lb. and the other 9 lb., are in equilibrium when they are connected by a string passing over a smooth pulley and completely immersed in water. If the specific gravity of the lighter body be 10, determine that of the heavier.

52. The weights of a mixture of quartz (sp. gr. 2·6) and gold (sp. gr. 19·4) in air and water are as 12 : 11. Compare the weights of the quartz and gold.

53. A square piece of wood of uniform thickness floats in water with two of its sides vertical and with $\frac{14}{15}$ of its surface area immersed. Find how deep it would sink when floating in a similar position in a fluid whose specific gravity is 1·4. *Show that if it floats with one diagonal horizontal* in a fluid composed of equal volumes of the fluid and water, then the depth of that diagonal below the surface will be one-sixth of its length.

54. Show that if the apparent weight of a body suspended in a liquid composed of volumes of two liquids which mix, without change of volume, is equal to the *arithmetic mean between its apparent weights when suspended* successively in the two liquids, the volumes of the two liquids are equal.

55. A uniform rod 12 inches long floats vertically with ·9 of its surface immersed in a cylindrical vessel containing water. If alcohol (sp. gr. ·8) be now poured on the water to the depth of 6 inches, show that the upper surfaces of the rod and alcohol will coincide.

56. A cubical block of wood (sp. gr. ·6) whose edge is 1 foot floats, with two faces horizontal, down a fresh-water river and out to sea, where a fall of snow takes place,

causing the block to sink to the same depth as in the river. If the specific gravity of the sea water be 1·025, and the weight of a cubic foot of fresh water 1000 ozwt., show that the weight of snow on the block is 15 ozwt.

57. A mass of 14 lb. of some substance seven times as specifically heavy as water is suspended from the hand by a string in a bucket of water. It is required to determine how much weight the string has to bear, and what changes are produced in the pressure at any point in the bottom of the bucket, in the force of the water on the bottom, and in the thrust on the floor. Which of these things can you determine, and which require that you should also know the area of the bottom of the bucket and the increase in the depth of the water?

58. A lump of metal of specific gravity 9·3 floats partly in oil of specific gravity ·9 and partly in mercury of specific gravity 13·5. What portion of its volume is in each?

59. A bucket full of water hangs by a cord. If a stone is dropped into the bucket, find the change in the pull along the cord. If a cork be dropped in the water, what is the change in the pull?

60. A uniform rod is suspended by two vertical strings attached to its extremities, and half of it is immersed in water. If the specific gravity of the rod be 2·5, the tensions of the strings will be as 9 : 7.

61. If a triangular lamina float with its plane vertical and one angular point in the surface of a fluid, prove that the opposite side will be vertical.

62. A cubic foot of water weighs 1000 oz. and a cubic foot of oil 840 oz. Oil is poured on water without mixing, and a sphere, the volume of which is 36 cubic inches and weight $19\frac{1}{8}$ ozwt., is placed in the mixture. How many cubic inches will be in each?

63. A block of oak whose specific gravity is 1·2 and weight 6 lbwt. is supported by a string, which cannot bear a force of more than 1·5 lbwt., in a barrel partly filled with

water, in which the block is wholly immersed. Fluid whose specific gravity is ·7 is now poured into the barrel, so as to mix with the water, until it is filled. Show that the string will break if the barrel be less than two-thirds full of water.

64. A lump of metal of specific gravity 10·5 floats partly in oil of specific gravity ·85 and partly in mercury of specific gravity 13·5. What portion of its volume is in each?

65. An iron ball of 12 lbwt. floats in mercury covered with water. Find the weights in each liquid.

66. A rod of uniform section is formed partly of platinum (sp. gr. = 21) and partly of iron (sp. gr. = 7·5), the platinum portion being 2 inches long. What will be the length of the iron portion when the whole floats in mercury (sp. gr. = 13·5) with one inch above the surface?

67. Eight cubic inches of lead whose specific gravity is 6, and a certain bulk of platinum whose specific gravity is 21, are connected by a fine thread passing over a smooth pulley and rest in equilibrium, when both bodies are wholly immersed in water. If the water be removed, by how much must the volume of the lead be diminished to preserve equilibrium?

68. A uniform beam AB, capable of turning about a fixed axis at the extremity A, rests with the portion BC immersed in water. Prove that the specific gravity of the beam is $1 - \left(\frac{AC}{AB}\right)^2$, and that the force on the fixed axis is to the weight of the fluid displaced in the ratio $AC : AB$.

69. A uniform rod 6 feet long can move about a fulcrum 1 foot above the surface of water. In the position of equilibrium 4 feet of the rod are immersed. Find its specific gravity.

70. The extremities of a uniform rod are attached by strings to a peg, one string being three times the length of the other. Three-quarters of the rod is immersed in water,

and the system rests with the strings equally inclined to the vertical. Show that the specific gravity of the rod is $\frac{5}{8}$.

71. If a parallelogram can float in a fluid with its plane vertical and two opposite sides horizontal, it must be rectangular.

72. A solid is placed in the upper cup of a Nicholson's hydrometer, and it is found that 12 grs. are required to sink the instrument to a certain depth. When the solid is in the lower cup, 16 grs. are required; and when the solid is removed, 22 grs. are required. What is the specific gravity of the solid?

73. The stem of a hydrometer is divided into 100 graduations, beginning from the top. When it is placed in a fluid of specific gravity 1·5, the surface of the fluid is at the graduation 20; when in a fluid of specific gravity 1·6, it is at the graduation 56. What is the specific gravity of a fluid of which the surface is at the graduation 96?

74. A Nicholson's hydrometer weighs 8 oz. The addition of 2 oz. to the upper pan causes it to be sunk in one liquid to the marked point, while 5 oz. are required in the other liquid. Compare the specific gravities.

75. In Nicholson's hydrometer, if specific gravities of the weights is 8, what weight must be placed in the lower pan to produce the same effect as 2 oz. in the upper pan?

76. A piece of copper weighs 11·378 gmswt. in air and 10·101 gmswt. in water. Find the specific gravity of copper.

77. A piece of wax weighs 26·653 gmswt. in air; the weight of a sinker is 11·378 gmswt.; weight of sinker in water is 10·101 gmswt.; weight of combination in water is 9·163 gmswt. Find the specific gravity of wax.

78. Weight required to sink Nicholson's hydrometer to the mark is 8·345 gmswt.; weight required to sink the hydrometer when a piece of sulphur is placed in the upper pan is 2·539 gmswt.; weight required when the sulphur is in the lower pan is 5·462 gmswt. Find the specific gravity of sulphur.

79. Weight of Nicholson's hydrometer is 11·265 gmswt.; weight required to sink the hydrometer to the mark in water is 8·345 gmswt.; weight required to sink the instrument in a solution of salt is 9·875 gmswt. Find the specific gravity of the solution.

80. The crown of Hiero, with an equal weight of gold and an equal weight of silver, were all weighed in water. The crown was found to lose $\frac{1}{14}$, the gold $\frac{4}{77}$, and the silver $\frac{2}{21}$ of the common weight. In what proportion were the gold and silver mixed in the crown?

81. Find the weight of a hydrometer which sinks as deep in rectified spirits of specific gravity ·866 as it sinks in water when loaded with 60 grs.

82. The weight of a piece of wood in air is 4 lbwt., of a piece of lead in water 4 lbwt., and of lead and wood together in water 3 lbwt. Required the specific gravity of the wood.

83. A hollow copper sphere, whose internal diameter is 2 feet, just floats in water. Find its thickness when the specific gravity of the copper is 8·8. [The volume of a sphere is nearly $4\frac{1}{5}$ times the cube of its radius.]

84. One body weighs 10 lbwt. in vacuo, and another 5 lbwt. in water. Their volumes are 48 and 72 cubic inches respectively. Compare their specific gravities.

85. If the specific gravity of iron, alcohol, and water be 7·2, ·8, and 1 respectively, find the internal radius of a spherical shell of iron ·1 inch thick which when filled with alcohol will just float in water.

86. A cylinder placed with its axis vertical in a fluid (sp. gr. $= S$) floats with an mth part immersed. When placed in another fluid (sp. gr. $= S'$) it rests with an nth part immersed. Find the part immersed when placed in a mixture composed of a parts of the first and b parts of the second fluid.

87. A sphere lighter than equal volume of water has its centre of gravity not coincident with its geometrical centre. State in what position it will float if put in water, and explain why it takes that position.

88. The weight required to sink a Nicholson's hydrometer is 1250 grswt. A small substance is placed in the upper scale-pan, and it is found that 530 grains are needed to sink the instrument to the standard point; but when the substance is put into the lower pan, 620 grains are required. What is the specific gravity of the substance?

89. A piece of wood exactly counterpoises a cubic inch of brass. What is its volume, supposing the specific gravity of the wood to be 0·9, that of brass 8·1, and that 800 cubic inches of the air, in which the weighing takes place, are as heavy as a cubic inch of water at the standard temperature?

90. A common hydrometer is found to sink to points marked *A*, *B*, *C* on its stem when placed in water and in two liquids respectively. The specific gravity of the latter liquid is ·9. If *B* is half-way between *A* and *C*, what is the specific gravity of the former liquid?

91. The stem of a common hydrometer is divided into 10 equal parts. The specific gravity corresponding to the lowest mark is 1, to the highest is ·8. Find the specific gravities corresponding to each of the other marks.

92. If a brass rider of 5 gmwt. be attached to the bottom of the hydrometer in Question 91, what will be the specific gravities corresponding to the 11 marks on the stem if the hydrometer weighs 15 gm.? (Sp. gr. of brass = 8.)

CHAPTER V.

ON GASES.

1. If a quantity of gas be placed in a closed vessel of any size it will extend itself to all parts of the vessel. This capacity of expanding indefinitely is the peculiar property of a gas by which it is distinguished from a liquid. If a liquid be placed in a vessel it will acquire the form of the part of the vessel it occupies, but there will be no appreciable change in its volume.

The most important gas is the air forming our atmosphere, and in discussing the properties of gases it is convenient to consider those of the air.

It was for a long time thought that air had no weight, and it was not until 1650 that the fact of air having weight was experimentally demonstrated by Otto Guericke. That **air has weight** may be proved by the following experiment: Take a large glass globe, and exhaust the air from it by means of an air pump (described in the next chapter). Weigh the globe, and then allow the air to enter, and weigh it again. It will be found that the globe is heavier when full of air than when empty, and the difference is the weight of the air the globe contained.

If the globe is 20 cm. diameter, it will contain about 4188 cc. of air, and since the average specific gravity of air is ·0013, the weight

of air the globe contains will be 5·44 gmwt. The weight of a cubic foot of air is 1·3 ozwt. The weight of air in a room 40 feet long, 30 feet broad, 15 feet high will be about 13 cwt.

Since a fluid exerts pressure at every point of a surface with which it is in contact, it follows that a gas will exert pressure at every point of the vessel within which it is contained. That the **air exerts pressure** is proved by the following experiments:

(i) If a tumbler be inverted and depressed vertically in a vessel containing water, it will be found that the surface of the water in the tumbler is below the surface of the water in the vessel. This difference of depths must be due to the pressure of the air in the tumbler.

(ii) If a glass be filled with water, and then carefully closed by a card and inverted, the pressure of the air will support the card and the water above it.

(iii) If a bladder containing only a small quantity of air be tied at the neck and placed under the receiver of an air pump, as the air is exhausted from the receiver, the air will expand within the bladder and inflate it.

2. **The Pressure of the Atmosphere.** The theorems of Chaps. III. and IV. have been proved for liquids, but only a slight modification is required to apply them to gases. In a gas as in a liquid the pressure is the same at all points of a horizontal plane; but as a gas expands indefinitely when the pressure is diminished, the density of the gas at the free surface will be very small, so that the free surface will not be clearly defined. In fact, there will be no permanent

free surface except in the case of the atmosphere. It follows from the consideration of the equilibrium of a vertical cylinder of gas, the principal section of which is the unit area, that the difference of pressures at two points in the same vertical is equal to the weight of this fluid cylinder. This difference of pressure will often be very small compared with the magnitude of each pressure. For example, in the case of the atmosphere (the specific gravity of which at the level of the sea being about ·0013), the difference of pressure due to a difference of 12 feet in height is only $\frac{1}{10}$ ozwt. per square inch, whilst the pressure per square inch at the level of the sea is 263 ozwt. When a gas is contained in a vessel of ordinary size, the pressure may generally he considered the same at each point, because the variation is small. This pressure is called the **Pressure of the Gas**, and it is due to the property of the gas, in virtue of which it always tends to expand itself. When a large volume of gas is considered, as in the case of the atmosphere, the change of pressure due to a great difference in height becomes important.

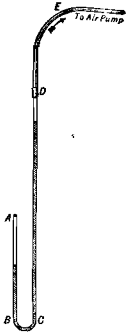
According to the Kinetic Theory of Gases, the Pressure of a Gas is due to the motion of the molecules of the Gas. In fact, the molecules of every body are in a state of motion amongst themselves. In solids the distance a molecule can travel is exceedingly small, and the forces acting between two molecules is very great, comparable indeed with the forces acting between parts of the same molecules. In liquids the distance a molecule can travel before meeting another molecule is much greater than in solids, and the forces acting between the molecules are smaller. The fact that one liquid can diffuse into another, and that difference of temperature will cause circulation in a liquid, shows that a molecule of a liquid

can wander from place to place in its mass, and is not compelled to move about a mean position. In gases the molecules have still greater freedom, and the forces acting between them are much less than in liquids. The average velocity of the molecules of a gas at ordinary temperatures is very great. For hydrogen at 0° C. it is 6190 feet per second, and for oxygen at the same temperatnre it is 1550 feet per second. A gas, therefore, very quickly alters its volume, and in order to keep it within a definite space, it must be confined by a boundary capable of resisting the motion of the molecules. Against this boundary the gas keeps up an incessant series of blows, and it is the continuous action of these blows which cause pressure. The pressure of a gas is therefore due to the motion of the molecules, and provided that no other forces act upon the molecules, the pressure can be measured by the rate of change of the momentum of the molecules which strike a square centimetre of the boundary. A gas is always in a condition in which it is exerting a force tending to change its volume; it is always in a state of stress. On this account gases have been called *elastic fluids*.

To measure the pressure of a gas a gauge can be used such as is described in Chap. III. (pp. 64, 65). The following experiment will illustrate the method of finding the pressure of a gas, and will suggest a method of finding the pressure of the atmosphere:

Make a U tube of glass, with one arm about 33 inches long and the other about 18 inches long. Pour mercury into the tube until it nearly fills the shorter branch *AB*. Now connect the long branch *CD* to an air pump by means of stout india rubber pressure-tubing *E*. Before any of the air is exhausted from the long arm the surfaces of the mercury in each branch will be level. This shows that the pressure of the air in *CD* is equal to the pressure of the atmosphere. Now exhaust some of the air from *CD*. There will imme-

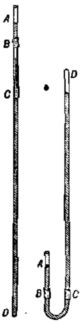
diately be a difference of level, showing that the pressure of the air in *CD* is less than that of the atmosphere. As the air is exhausted more and more from *CD*, the difference of level increases. A limit will soon be reached, beyond which it is impossible to exhaust the air further. With a good air pump the difference of level attained may be about 29 inches, in which case the pressure of the atmosphere is greater than that of the air by about 29 inches of mercury. Now, if it were possible to exhaust *all* the air from *CD*, and so reduce the pressure in *CD* to nothing, then the difference of level would measure the pressure of the atmosphere.



In order to exhaust all the air from *CD*, the experiment may be modified in the following way: Fit tightly on to *D* a piece of flexible india rubber tubing, and provide it with a clip. Now tilt the tube until the mercury fills the long branch and part of the rubber tubing. This will drive all the air out of *CD*. To prevent it returning when the tube is restored to the vertical position, firmly clip the india rubber tubing. Now place the tube in a vertical position, and there will be found to be a difference of level of about 29 to 30 inches. Since there is no air in *CD*, the difference of level is due to the pressure of the atmosphere and

hence the pressure can be measured in inches of mercury.

The air in the long branch can be exhausted somewhat more easily by the following method: Take a piece of glass tubing *CD* (32 inches long) and close it at one end. Join it to a short piece of glass tubing by means of india rubber tubing. Now, holding the whole vertically, fill it with mercury up to within 3 inches from the top, taking care not to allow air bubbles to remain in the mercury. Now carefully raise the end *D*, keeping *AB* vertical, until *CD* is also vertical, thus forming a U tube. The mercury in *CD* will fall, leaving a vacuum at *D*, and there will be found to be a difference of level due to the pressure of the atmosphere. If we could be certain that no air would leak in at the joints, this apparatus could be used to find the pressure of the atmosphere in inches of mercury at any time. It is in fact a particular form of *barometer.*

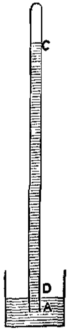


Torricelli, in 1643, was the first to determine experimentally the pressure of the atmosphere, and he constructed for the purpose an instrument which is now called the **Barometer.**

To construct a mercurial barometer.

Take a glass tube *CA* not less than 32 inches long, closed at one end and open at the other. Fill the tube with mercury and close the open end *A*, then invert

the tube and place it vertically with the end A in a vessel containing mercury. Now open the end A. The mercury in the tube will then be seen to fall, leaving a vacuum at the top, until its surface is at a certain height above the surface of the mercury in the vessel. This height, which is called the height of the *barometer, depends upon the pressure of* the atmosphere, and varies in England, at sea level, between 29 and 31 inches.



To show that the height of the barometer is a measure of the atmospheric pressure.

Let D be a point where the tube meets the surface of the mercury in the vessel.

(1) The pressure at any point on the surface of the mercury in the vessel is equal to the pressure of the atmosphere.

(2) Since there is no pressure on the surface of the mercury in the tube, the pressure at any point of the mercury within the tube at the same level as $D = w \, . \, CD$ (Prop. II. p. 50).

But the pressure is the same at all points of a fluid in the same horizontal plane (Prop. I. p. 49).

Therefore pressure of the atmosphere $= w \, . \, CD$; and the pressure of the atmosphere is proportional to the height of the barometer.

Note 1.—Every liquid gives off vapour at its surface, so that the space in the tube above C will not be quite a vacuum, but will

contain the vapour of mercury. This space in the tube is called the *Torricellian vacuum.*

NOTE 2.—The height of the barometer is independent of the shape of the barometer tube, for the pressure varies as the depth below the free surface, whatever be the shape of the vessel (Cor. I. p. 56). It follows, therefore, that if the barometer be inclined, the surface of the mercury at C will remain at the same height above the mercury in the vessel; and that it is not essential for the tube to be of uniform bore. The tube could be of any spiral form whatsoever; but when the vertical height is required, the advantage of using a straight vertical tube is obvious.

NOTE 3.—Any other liquid could be used besides mercury; but the mercury barometer possesses many advantages (see Note 6). Amongst these is its convenient height.

Ex. 1.—If the height of a mercury barometer is 30 inches, what is the pressure of the atmosphere (per square inch), and what is the corresponding height of a water barometer?

The height of mercury barometer = 30 inches.

∴ force of atmosphere on a square *inch* = weight of 30 cubic inches of mercury = $14\frac{3}{4}$ lbwt. nearly (Ex. 1, p. 20).

∴ pressure of the atmosphere = $14\frac{3}{4}$ lbwt. per sq. inch.

To find the height of the water barometer.

Since mercury is 13·596 times heavier than water, the number of cubic inches of water required to exert the same force on a square inch will be 13·596 times the number of cubic inches of mercury required; and therefore the height of the water barometer will be 13·596 times the height of mercury barometer.

∴ height of water barometer = 13·596 × 30 inches;
= 407·88 inches;
= 33 ft. $11\frac{22}{25}$ in., nearly 34 ft.

Ex. 2.—If the pressure of the atmosphere is $14\frac{3}{4}$ lbwt. per square inch, what will be the height of the glycerine barometer? (Sp. gr. of glycerine 1·26.)

The force is $14\frac{3}{4}$ × 144 lbwt. on the square *foot*. The question is, how many cubic feet of glycerine will weigh $14\frac{3}{4}$ × 144 lbwt.?

Let V cubic feet of glycerine weigh $14\frac{3}{4} \times 144$ lbwt.

$$\therefore 14\tfrac{3}{4} \times 144 \times 16 \text{ ozwt.} = V \times 1{\cdot}26 \times 1000 \text{ ozwt.}$$

$$\therefore V = \frac{14\frac{3}{4} \times 144 \times 16}{1{\cdot}26 \times 1000} \text{ cubic feet;}$$

$$= 26\tfrac{102}{105} \text{ cubic feet;}$$

$$= 27 \text{ cubic feet nearly.}$$

Therefore the height of the glycerine barometer is 27 feet nearly. When the pressure is $14\frac{3}{4}$ lbwt. per square inch, the height of the water barometer is 34 ft. nearly. Therefore the height of the glycerine barometer will be $\frac{34}{1{\cdot}26}$ feet $= 27$ feet nearly.

Ex. 3.—If the height of the mercury barometer is 760 mm., what is the pressure of the atmosphere in dynes per sq. cm.? (The weight of 1 gm. at the particular place being 981·17 dynes.)

The pressure of the atmosphere $=$ weight of 76 cc. of mercury per sq. cm.

$$= 76 \times 13{\cdot}596 \text{ gmwt. per sq. cm.}$$

$$= 76 \times 13{\cdot}596 \times 981{\cdot}17 \text{ dynes per sq. cm.}$$

$$= 1013800 \text{ dynes per sq. cm. nearly.}$$

Hence the pressure of the atmosphere when the barometer is 760 mm. is approximately a million dynes per sq. cm.

EXERCISES.

1. The height of the mercury barometer is 28 inches, and the difference of level in a syphon gauge is 25 inches. What is the pressure of the gas?

2. The height of the water barometer is 33 feet. What is the corresponding height of a mercury, and of a glycerine barometer? If the mercury barometer falls $\frac{1}{10}$ inch, how much do the water barometer and the glycerine barometer respectively fall?

3. The glycerine barometer stands at 27·5 feet. Find the pressure of the atmosphere in lbwt. per sq. inch.

4. A small quantity of ether is admitted into the Torricellian vacuum of a mercury barometer, and the barometer sinks from 29·5 inches to 14·9 inches. What is the pressure of the ether vapour in inches, and in mm. of mercury, and in gmwt. per sq. cm.?

5. How high will the mercury barometer stand if the pressure of the atmosphere is 14 lbwt. per sq. inch?

6. If the glycerine barometer falls 2 feet, what is the corresponding change in the force on a square foot exposed to the atmosphere?

7. If the mercury barometer falls 1 inch, what is the corresponding fall in the water barometer?

NOTE 4.—*Precautions necessary in constructing a barometer.*

(i) Care should be taken to use *pure* mercury, and to heat the mercury before using it, in order to drive away any water it may contain; for the water will rise into the vacuum and form vapour, which will exert a downward force on the column and diminish its height. For the same reason care should be taken when inverting the tube not to allow any globules of air to rise into the vacuum.

To avoid this source of error, the tube is best filled by the application of heat. The open end of the tube is placed in a vessel of pure boiled mercury, and the air is driven from the tube by carefully heating it with a Bunsen burner. If the tube be inclined to the vertical, the pressure of the atmosphere will drive the mercury up the tube until it is completely filled.

(ii) If the tube is graduated it must be placed vertically, otherwise the height read from the graduations will be greater than the true vertical height. The pressure within the tube at D depends simply on the vertical depth below the surface of the mercury within the tube. The pressure being the same, this surface will be in the same horizontal plane whatever be the inclination of the tube.

NOTE 5.—*Corrections to be applied to the apparent height of the barometer.*

(i) Mercury expands on increase of temperature, so that even if the pressure of the air is the same on any two days, the mercury will be higher in the tube on the warmer of the two days.

If h be the height of the barometer at 0° C., and h' the height at t° C., then it can be proved that the height of mercury corresponding to the same pressure increases by $\frac{1}{5550}$ of its length for 1° C., and therefore it will increase $\frac{t}{5550}$ of its length for t° C.

$$\therefore h = h' - \frac{t}{5550} h'.$$

If the temperature is 15° C. and the observed height 30 inches, it follows that the correction is $\frac{15 \times 30}{5550}$ inch $= \frac{9}{111}$ inch, and therefore the height at 0° C. is $29\frac{102}{111}$ inch. The correction is very small.

(ii) The vapour of mercury will exert a downward force on the surface at C, and diminish the height. Its effect can be determined experimentally, and it is found to increase with the temperature. If the temperature is t° C., the pressure of mercury vapour within the limits of the temperature of the air is $\cdot 002t$ mm. of mercury.

(iii) *As the mercury rises in the tube it falls in the vessel, so* that there is no fixed zero from which to measure the height of the barometer. The tube might be graduated from its lowest point, then the difference of the readings at the two surfaces would give the height of the barometer. In some of the best instruments there is an apparatus for moving the vessel vertically, so that the surface of the mercury in it can be brought to a fixed zero before reading the graduation. This is convenient, because only the height of the mercury in the tube need be read, and this can be done with great accuracy if the scale is provided with a vernier.

(iv) A correction must be made for the variation in the length of the scale, with changes of temperature. If the scale has been graduated true at 0° C., and the apparent height at t° C. is h, then the true height is $h + \alpha th$, where α is the linear coefficient of expansion of the scale.

(v) It will be noticed that the surface of the mercury in the tube is drawn downward at its edges. This is especially noticeable if the bore of the tube is very narrow, about ·25 cm., when the surface will appear almost spherical. It is also found by experiment that if an open tube of very fine (capillary) bore be dipped into mercury, the surface of the mercury inside the tube will be depressed below the general level outside. This depression is due to the capillary action between the glass of tube and the mercury. If the bore of a barometer tube is small—less than 1 cm.—the effect of capillary action will be to draw down the mercury by an amount depending upon the diameter of the bore. If the bore is ·5 cm. diameter the depres-

sion is about ·025 mm.; if the bore is ·25 cm. diameter the depression is about ·1 mm., and generally the depression is inversely *proportional to the square of the diameter of the bore*. In accurate reading of the height of a barometer, the diameter of the bore of the tube must be known, and a correction made for capillarity.

NOTE 6.—*Reasons for using mercury in barometers.*

(i) Mercury is the heaviest liquid, and consequently the height of the barometer is the least possible. The water barometer would require a tube 34 feet long.

(ii) The vapour pressure of mercury at the average temperature of the atmosphere is less than any other liquid (with perhaps a few exceptions, *e.g.* sulphuric acid), and therefore a better vacuum can be obtained.

(iii) Mercury expands by increase of temperature less than any other liquid.

(iv) Mercury does not wet glass, and therefore it moves easily in the tube.

3. Boyle's Law. If a gramme of gas be introduced, into a closed vessel, it will expand and exert a certain force upon every square inch throughout the vessel. If a second gramme be introduced, and *if we assume that its effect is not altered by the presence of the first gramme,* then it will exert an equal force upon every square inch throughout the vessel, so that the pressure will now be doubled. If a third gramme be introduced, the pressure of the gas within the vessel will now be three times the pressure of the first gramme. Hence the pressure of a gas in a vessel varies as the mass of the gas. It also follows that the pressure of a gas in a given space (*e.g.* in a cubic foot) varies as the mass of the gas within the space, or the pressure of the gas varies as its density.

If the volume of the vessel be doubled, the gas will expand and fill the vessel, and therefore the mass of the gas in a cubic foot will be halved, *i.e.* the density will

be halved; and generally the density of a given mass varies inversely as its volume. Therefore the pressure of a given mass of gas varies inversely as its volume. The temperature is considered constant, so that there is *no change of volume due to* change *of* temperature.

A gas has been defined as a fluid, the volume of which increases as the pressure upon it is reduced. If the gas is such that there are no forces acting between the molecules except during collisions (in such case the effect of one gramme in causing pressure will not be altered by the presence of another gramme), then, if the pressure is reduced to half, the volume will be doubled. This is probably the greatest increase of volume possible for the same change of pressure, but there are many gases in which the increase of volume would be less. Again, if the volume of a gas is reduced to half, the density will be doubled, and the number of impacts on a sq. cm. will be doubled; and therefore, if one molecule does not act upon another except during collisions, the pressure will be doubled; for so long as the temperature remains unaltered, the average velocity of the molecules is the same for all densities. A gas in which the volume increases inversely as the pressure is called a **Perfect Gas.** If a gas is such that a slight decrease of volume without diminution of temperature causes a part of it to be condensed to a liquid, it is called the **Vapour** of the Liquid. If the volume of a vapour is increased, the pressure will remain unchanged until all the liquid is evaporated; after that, as the volume is increased, the pressure will diminish, but at first not so rapidly as a Perfect Gas. As the volume becomes larger and larger, the gas more and more nearly behaves as a Perfect Gas. Many gases, such as Air, Hydrogen, Oxygen, Nitrogen, obey the Law of a Perfect Gas very nearly under all pressures to which they have been subjected at ordinary temperatures. They are called **Permanent Gases.** It is found, however, that carbonic acid gas, if kept at a temperature below 31·1° C., can be liquified by increase of pressure. The study of vapours belongs to the subject of heat; it was, however, necessary to state as much as this in order to warn students that all that is said about gases in

this chapter refers only to gases which obey the Law of Perfect Gas within the limits of the given pressures and volumes.

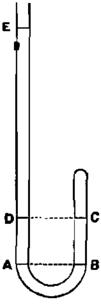
The above relation between the pressure and volume of a Perfect Gas was discovered by Boyle (1662) and by Mariotte (1679), and is known in England as Boyle's Law, and on the continent as Mariotte's Law.

BOYLE'S LAW.—**If the temperature be kept constant, the pressure of a given mass of gas varies inversely as its volume.**

It may also be stated thus:

If the temperature be kept constant, the pressure of a gas varies as its density.

This is the first of the two "Gaseous Laws." The other law states the relation between the volume and temperature of a gas when the pressure is kept constant. It is considered in books on Heat. A brief account of it is given in the Appendix.



Experimental proof of Boyle's Law.

Take a U tube having one branch shorter than the other. The end of the shorter branch is closed, the other end is open. Place the tube vertical and pour in a small quantity of mercury, and adjust it so that its surfaces at A and B are in the same horizontal plane. Then the pressure of the air in the

short branch must equal the pressure of the air at A, *i.e.* equal the pressure of the atmosphere. Now pour in mercury until the volume of the air in the short branch is reduced to half its original volume. Let C and E be the surfaces of the mercury in the two branches, and suppose D is in the same horizontal plane as C. Then it will be found that DE is equal to the height of the mercury barometer.

Hence the pressure at D = pressure of the atmosphere at E + the pressure due to the depth DE.

Therefore the pressure at D = twice pressure of the atmosphere. But pressure of air in short branch = pressure at D.

Therefore the pressure in short branch = twice pressure of atmosphere = twice the original pressure.

Therefore, when the volume of the air in the short branch is reduced to half its first volume, the pressure will be twice the original pressure.

In a similar manner it can be shown that, if the volume be reduced to *one-third* its original volume, then the pressure will be *three times* the original pressure. And generally the pressure can be shown to vary inversely as the volume.

COR. I.—If Π is the pressure of a given mass of gas when its volume is V, and Π' the pressure when the volume is V', then Boyle's Law states that

$$\Pi : \Pi' :: V' : V, \text{ or } \Pi V = \Pi' V'.$$

$$\therefore V' = \frac{\Pi}{\Pi'} V \text{ and } \Pi' = \frac{V}{V'} \Pi.$$

COR. II.—Boyle's Law has been established by experiment, and we have seen that it expresses the fact that **the pressure of a gas is unaltered by the presence of any other gas.** To find the pressure of a mixture of gases it is only necessary therefore to find the pressure of each gas, and the sum of the pressures will be the pressure of the mixture.

Thus, if a volume V' of a gas at pressure Π' be mixed with a volume V'' of another gas at pressure Π'' in a vessel of volume U, to find the pressure of the mixture,

(1) Pressure of the first gas will be $\frac{\Pi' V'}{U}$;

(2) Pressure of the second gas will be $\frac{\Pi'' V''}{U}$.

Therefore, if Π is the pressure of the mixture,

$$\Pi = \frac{\Pi' V'}{U} + \frac{\Pi'' V''}{U};$$

or

$$\Pi U = \Pi' V' + \Pi'' V''.$$

Ex. 1.—If 50 cubic inches of air at a pressure of 15 lbwt. per sq. inch be placed in a vessel the volume of which is 70 cubic inches, what will be the new pressure?

When the volume is 50 cubic inches, the pressure is 15 lbwt. per sq. inch.

If the volume is reduced to 1 cubic inch, the pressure would become 15×50 lbwt. per sq. inch.; and if the volume is now compressed to 70 cubic inches, the pressure would become $\frac{15 \times 50}{70}$ lbwt. per sq. inch.

Hence the new pressure $= 10\frac{5}{7}$ lbwt. per sq. inch.

Ex. 2.—When the volume of a gas is 50 cubic inches, the pressure is 15 lbwt. per sq. inch. What must its volume

be altered to in order to increase its pressure to 40 lbwt. per sq. inch?

When the pressure is 15 lbwt. per sq. inch its volume is 50 cubic inches.

To reduce the pressure to 1 lbwt. per sq. inch, the volume must be 50×15 cubic inches.

∴ to make its pressure 40 lbwt. per sq. inch

the volume must be reduced to $\frac{50 \times 15}{40}$ cubic inches.

$$\therefore \text{ the new volume} = \frac{50 \times 15}{40} \text{ cubic inches.}$$

$$= 18\tfrac{3}{4} \text{ cubic inches.}$$

Ex. 3.—In the U tube, used to prove Boyle's Law, mercury is poured in so that the surfaces in each branch are at the same level AB. The air in the short branch is found to occupy 30 cm. of the tube, and the height of the barometer is 760 mm. More mercury is now poured in, and height of the surface in the open arm above that in the short arm is measured and found to be 280 mm. What will be the new pressure of the air in the closed branch, and what length of tube will it occupy?

The pressure of the air in the short arm at first = the pressure of the atmosphere = pressure due to 160 mm. of mercury.

The new pressure of the air in the short arm
= pressure of atmospbere + pressure due to 280 mm. of mercury;
= pressure due to 760 + 280, *i.e.* 1040 mm. of mercury.

Now the air occupies 30 cm. of tube at pressure of 760 mm. of mercury.

∴ it will occupy 30×760 cm. at a pressure of 1 mm. of mercury.

" $\frac{30 \times 760}{1040}$ cm. at a pressure of 1040 mm. of mercury.

the length of tube occupied by the compressed air

$$= \frac{30 \times 760}{1040} \text{ cm.}$$

$$= 21{\cdot}92 \text{ cm.}$$

Ex. 4.—*If 20 cubic inches of air at 15 lbwt. per sq. inch pressure and 30 cubic inches of air at 10 lbwt. per sq. inch pressure be placed together in a vessel the volume of which is 50 cubic inches, find the pressure in the vessel.*

(i) The pressure of the first quantity of air if placed in a vessel of 50 cubic inches capacity will be $\frac{20}{50} \times 15$ lbwt. per sq. inch, *i.e.* 6 lbwt. per sq. inch.

(ii) The pressure of the second quantity of air placed in the same vessel of 50 cubic inches capacity will be $\frac{30}{50} \times 10$ lbwt. per sq. inch, *i.e.* 6 lbwt. per sq. inch.

Therefore, since the pressure of one gas is unaltered by the presence of the other, the pressure of the mixture is the sum of the pressures of each gas, *i.e.* 12 lbwt. per sq. inch.

Ex. 5.—*If the volume of the Toricellian vacuum is 5 cubic inches, the section of the tube 1 square inch, and the height of the barometer 30 inches, what will be the volume of air at atmospheric pressure which, if allowed to rise into the vacuum, will lower the height of the barometer 2 inches?*

The pressure of the atmosphere (per square inch) = weight of 30 cubic inches of mercury.

The pressure of the air in the tube (per square inch) = weight of 2 cubic inches of mercury.

The volume of the air is 7 cubic inches.

∴ the volume of the air at atmospheric pressure

$$= \frac{2}{30} \times 7 \text{ cubic inches.}$$
$$= \frac{7}{15} \text{ cubic inch.}$$

EXERCISES.

1. The pressure of a gas is 14 lbwt. per sq. inch when the volume is 25 cubic feet. What is the pressure when the volume is 150 cubic feet?

2. The pressure of a gas is 25 lbwt. per sq. inch when

the volume is 80 cubic feet. What is the pressure when the volume is 3 cubic feet?

3. *Find the volume of a gas at 15 lbwt. per sq. inch pressure which occupies a volume of 350 cubic feet at 3 lbwt. per sq. inch pressure.*

4. Find the volume of a gas at 76 cm. of mercury pressure if it occupies a volume of 15 cc. at 85 cm. of mercury pressure.

5. Find the volume of a gas at 800 cm. of mercury pressure if it occupies a volume of 103 cubic inches at 95 cm. of mercury pressure.

6. If 30 cubic inches of air at 18 lbwt. per sq. inch pressure and 40 cubic inches at 10 lbwt. per sq. inch pressure be forced into a vessel of 15 cubic inches capacity, what will be the pressure of the mixture?

7. If *3000 cc. of air at 760 mm. of mercury pressure* and 5000 cc. of air at 850 mm. of mercury pressure be *placed into a vessel of 6000 cc. capacity,* what will be the pressure in the vessel?

8. If the two quantities of air in Question 7 were placed in a vessel of such a capacity that the pressure was 500 mm. of mercury, what must the volume of the vessel be?

9. In proving Boyle's Law, the differences of levels are found to be 12 mm., 60 mm., 240 mm., 375 mm., 485·5 mm., 835·0 mm., and the length of the tube occupied by the enclosed air was 350 mm. in the first case. If the height of the barometer was 765 mm., find the length of the tube occupied by the air in the other five cases.

10. A uniform barometer tube is filled with mercury to within 1·5 inches of the top. It is then closed, and inverted

into a basin of mercury. When the end is opened it is found that the mercury stands at 600 mm. If the height of the barometer is 760 mm., what is the length of the tube now occupied by the enclosed air?

11. A bladder contains 2 cubic inches of air at atmospheric pressure (34 feet of water). It is sunk in water to a depth of 100 feet. Find what its volume becomes.

12. A bladder contains 100 cc. of air at atmospheric pressure (760 mm. of mercury), and it is sunk to 30 feet in water. What will its volume become?

13. A bladder of air is found to occupy a volume of 150 cc. at a depth of 60 feet in water. What will its volume be at the surface? (Height of water barometer 33 feet.)

14. A bubble of air has a volume of ·1 cc. at a depth of 300 cm. in mercury. What is the upward force of the mercury upon it? It rises 100 cm. What is the new volume of the bubble, and what is the upward force of the mercury upon it?

15. A bladder contains 100 cc. of air at the pressure of the atmosphere (34 feet of water). What weight must be attached to it so as just to float it totally immersed near the surface of water? If the bladder is now depressed 68 feet, what will its volume become, and what will be the weight of water it will displace? If left to itself will it sink or rise?

Cor. III.—Boyle's Law states that the pressure of a gas varies as its density, or the density of a gas varies as the pressure, the temperature being kept constant. Hence, if the density of a gas is known at a given temperature and pressure, the density at any other pressure

can be found by Boyle's Law, provided the temperature is unaltered. It is convenient to have a standard temperature and pressure to which to refer the densities of gases. The standard temperature is the temperature at which ice melts, viz. 0° C. The **standard pressure** *is a pressure corresponding to a height of 76 cm. of mercury when the temperature of the mercury is* 0° C. This is about the average height of the barometer, and is equal to 29·922 inches. The standard pressure is therefore about 14·7 lbwt. per sq. inch, and is called an *atmosphere.* From Ex. 3, p. 132, it is found that the standard pressure at Paris, where the acceleration of a falling body is 981·17 cm. per sec. per sec., is 1013800 dynes per sq. cm. Since the weight of a body varies in different localities, the weight of 76 cc. of mercury will vary, and therefore the standard pressure as above defined is not constant. On this account it has been proposed to take as standard pressure a million dynes per sq. cm.

The densities of a large number of gases have been determined at standard temperature and pressure (S.T.P.). For example, the density of air at S.T.P. is ·0013 gm. per cc.; of hydrogen ·00008957 gm. per cc.; of oxygen ·00143 gm. per cc. By Boyle's Law the density at any other pressure can be determined.

If D gm. per cc. be the density at pressure of p units of pressure, and D' gm. per cc. be the density at pressure of p' units of pressure, then, by Boyle's Law,

$$p : p' :: D : D'.$$

$$\therefore D' = \frac{p'}{p} . D.$$

If s = sp. gr. of a gas at pressure p, and s' = sp. gr. of the gas at pressure p', then $s' : s :: p' : p$.

$$\therefore s' = \frac{p'}{p} s.$$

Ex. 1.—The specific gravity of air at the pressure of an atmosphere is ·0013. What is the specific gravity at pressure of $2\frac{1}{2}$ atmospheres?

$$\text{Sp. gr. at } 2\tfrac{1}{2} \text{ atmos} = \frac{2\frac{1}{2} \times \cdot 0013}{1} = \cdot 00325.$$

Ex. 2.—If $\frac{1}{10}$ oz. of hydrogen (the specific gravity of which is ·00009) at standard pressure be placed in a vessel of 3 cubic feet, what will be the new specific gravity and pressure?

If s = the new sp. gr. of the hydrogen, then 3 cubic feet of hydrogen at sp. gr. s weighs $\frac{1}{10}$ oz.

$$\therefore 3 \times s \times 1000 \text{ oz.} = \tfrac{1}{10} \text{ oz.}$$

$$\therefore s = \tfrac{1}{30000} = \cdot 0000\dot{3};$$

also $$\frac{\text{the new pressure}}{\text{standard pressure}} = \frac{\cdot 0000\dot{3}}{\cdot 00009} = \tfrac{10}{27}.$$

$$\therefore \text{the new pressure} = \tfrac{10}{27} \times \text{standard pressure.}$$

Ex. 3.—2000 cc. of hydrogen is collected at a pressure of 300 mm. of mercury. What is its density and mass? (Temperature 0° C.)

Density of hydrogen at 760 mm. of mercury = ·00008957 gm. per cc.

$\therefore$ density of hydrogen at 1 mm. of mercury $= \dfrac{\cdot 00008957}{760}$ gm. per cc.

$\therefore$ density of hydrogen at 350 mm. of mercury

$$= \frac{350 \times \cdot 00008957}{760} \text{ gm. per cc}$$

$$= \cdot 00004125 \text{ gm. per cc.}$$

$\therefore$ Mass of 1 cc. = ·00004125 gm.
Mass of 1000 cc. = ·04125 gm.
Mass of 2000 cc. = ·08250 gm.

Ex. 4.—·5 gramme of hydrogen is found to occupy a volume of 1500 cc. What is its pressure? (Temperature 0° C.)

Since ·5 gm. of hydrogen occupies a volume of 1500 cc., 1 gm. of hydrogen will occupy a volume of 3000 cc.

But 1 gm. of hydrogen at 760 mm. pressure occupies a volume of $\frac{1}{\cdot 00008957}$ *cc., i.e.* 11160 cc.

The question therefore is, what pressure will the gm. of gas exert when its volume is changed from 11160 cc. to 3000 cc.? By Boyle's Law this is found to be $\frac{11160}{3000} \times 760$ *mm., i.e.* 2827·2 mm.

EXERCISES.

[The density ratio (with hydrogen at S.T.P. as standard) of oxygen is 16, nitrogen 14.]

1. Find the mass of 100 cc. of hydrogen at 0° C. and 380 mm. pressure.

2. Find the mass of 3500 cc. of oxygen at 0° C. and 850 mm. pressure.

3. Find the mass of 6875 cc. of nitrogen at 0° C. and 25 mm. of mercury pressure.

4. Find the mass of 10 cc. of air at 0° C. and 4 mm. of mercury pressure.

5. Find the volume of 10 gm. of oxygen at 800 mm. of mercury pressure.

6. Find the volume of ·35 gm. of hydrogen at 10 mm. of mercury pressure.

7. Find the volume of 3·5 gm. of air at 765 mm. of mercury pressure.

8. At what pressure will a gramme of air occupy a volume of 1000 cc.?

9. At what pressure will ·5 gm. of oxygen occupy a volume of 655 cc.?

10. ·63 gm. of air is put into a vessel of 1500 cc. capacity. What pressure will it exert?

11. ·65 gm. of air is placed in a vessel and found to exert a pressure of 850 mm. of mercury. Find its volume.

12. The volume of a mass of air is 50000 cc., and the pressure is 4 mm. of mercury. What is the mass?

13. A body is found to displace ·6 gm. of air at 750 mm. of mercury pressure. What is the volume of the body?

4. The force of the atmosphere on every horizontal square inch being equal to the weight of the cylinder of air vertically above it, the upper end of which is in the free surface of the atmosphere, it is easily seen that the pressure of the atmosphere diminishes gradually from the level of the sea upwards. It follows from Boyle's Law that the density (and therefore the specific gravity*) of the atmosphere diminishes from the level of the sea upwards. The pressure, and therefore the density at any point, depends upon its height above the level of the sea, but to determine the relation between the pressure and the height requires the use of somewhat high mathematics. If the pressure is determined by means of a barometer, the density can be found by Boyle's Law. Thus, if the barometer stands at 15 inches, the reading at sea level being 30 inches, then, since the pressure is reduced to half, the density is also reduced to half. An approximate value

of the height above the sea may be determined if we assume that the difference of pressures at two points is equal to that due to a column of air, the density of which is uniform and equal to the mean of the densities at the two points.

Ex.—*The mercury barometer stands at* 30 *inches at sea level and at* 18 inches at the top of a mountain. Find an approximate value for the height of the mountain, given that the specific gravity of air at sea level is ·0013 and of mercury 13·5.

By Boyle's Law

$$\frac{\text{the sp. gr. at top of mountain}}{\cdot 0013} = \tfrac{18}{30} = \tfrac{3}{5}.$$

$$\therefore \textit{sp. gr. at top} = \tfrac{3}{5} \times \cdot 0013 = \cdot 00078.$$

$$\therefore \text{mean sp. gr.} = \tfrac{1}{2}\{\cdot 0013 + \cdot 00078\} = \cdot 00104.$$

The difference of pressures per square inch at the two points is equal to a weight of (30 − 18) cubic inches of mercury, *i.e.* of 12 cubic inches of mercury.

The problem therefore is to find what column of air of specific gravity ·00104 is equal in weight to a column of mercury 12 inches high.

Now,

length of column of air : length of column of mercury :: 13·5 : ·00104.

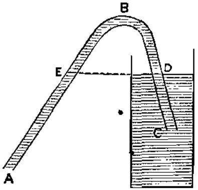
$$\therefore \text{length of column of air} = \frac{13\cdot 5}{\cdot 00104} \times 12 \text{ inches};$$

$$\doteq 12980 \text{ feet nearly}.$$

5. The following are practical illustrations of the principle of Boyle's Law:

(i) The **Siphon** is a bent tube, having both its ends open, and is used for drawing liquids out of vessels. It is filled with the liquid, and both of its ends are closed. It is then inverted with its end C in the liquid in the

vessel, and with its end A below the surface of the liquid in the vessel. The ends are then opened, and the liquid will run out in a continuous stream through the tube.



To explain the action of the siphon.

Let the arm BC meet the surface of the liquid in D, and let DE be a horizontal line.

Pressure of the liquid at E = pressure at D;
= pressure of the atmosphere.

Pressure of the liquid at A = pressure at E + pressure due to column AE.

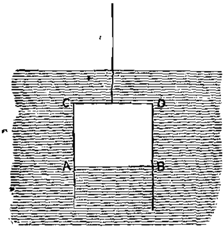
Therefore pressure of the liquid at A is greater than the pressure of the atmosphere at A.

Therefore the liquid in AB will fall, and form a vacuum at B: and if the height of B above the surface of the liquid in the vessel is less than the height of the liquid barometer, the liquid in BD will rise, and continue to flow down the tube BA until the vessel is empty, or until the height of B above the surface is equal to the height of the liquid barometer.

NOTE.—The greatest height over which a siphon will draw a liquid is the height of the liquid barometer. Thus water can be drawn over any height under 34 feet, and mercury over any height under 30 inches.

If the end *A* is above the level of the water in the vessel, the water will not run out of *BE*.

(ii) The **Diving Bell** is a large bell-shaped vessel made of metal, and open at the bottom. Its weight is



greater than that of the water it would contain. It is lowered by a chain, with its open end downwards, into water. As it descends the pressure of the water on the air within it increases, the air is compressed, and the water rises in the bell. As the air is compressed its pressure increases, and prevents the water rising too high in the bell, so that persons are enabled to descend with safety to considerable depths in water.

NOTE 1.—The bell is generally furnished with two tubes leading into it, so that fresh air can be driven in through one and the impure air drawn out through the other.

Note 2.—The pull of the chain = weight of bell — weight of water displaced.

As the bell descends the weight of water displaced decreases, and therefore the pull of the chain increases.

Note 3.—The diving bell can be of any shape and of any suitable material. It is frequently in the form of a flexible diving dress, somewhat fitting the diver, and loaded at the feet to make it sink. Air is forced in through one tube and exhausted through another. When the diver wishes to rise, air is forced in to expand the dress and increase the buoyancy of the whole.

Note 4.—In making the foundations for a bridge across a river the principle of the diving bell is used. A large cylindrical casing of metal is sunk vertically in position. The lower part of the casing is formed into an air-tight chamber, and above it is a second air-tight chamber, furnished with air-tight doors. Air is forced into the lower chamber at such a pressure as to overcome the pressure of the water due to the depth below the surface. In this chamber men work and excavate below the casing, which is thus gradually sunk until a solid base is reached. The inside of the casing is afterwards filled with concrete, and upon it one of the piers of the bridge is built. The casings for the foundations of the Forth Bridge are huge structures, 70 feet diameter at the bottom and 60 feet at the top, each weighing 400 tons when empty and 15000 tons when full of concrete. The air-tight working chambers are 7 feet high. The pressure of the air inside is sometimes as great as 33 lbwt. per sq. inch above atmospheric pressure, and the work of excavating is done by men specially trained to work under great pressure.

Note 5.—It may be noticed that if a rigid body floats on the surface of the water inside the bell, that as the bell descends the air is compressed, and its specific gravity increases, and consequently the buoyancy of the air increases, and the body rises a little out of the water.

Ex. 1.—If the water barometer stand at 34 feet, how deep must a bell be sunk so that the water may fill one-third of it?

Since the volume of the air in the bell is reduced to one-third its original volume, the pressure of the air in the bell is three times the original pressure, *i.e.* three times the pressure of the atmosphere. The pressure of the air within the bell has therefore been increased

by twice the pressure of the atmosphere. *And this increase of pressure is due to the depth of the surface of the water in the bell.* Therefore the surface of the water in the bell must be at a depth *equal to twice the height of the water barometer, i.e.* at a depth of 68 feet below the surface of the water outside the tube.

Ex. 2.—A tube 30 inches long, closed at one end, is depressed vertically with its open end downwards in water. Find the height the water rises in the tube when the surface of the water in the tube is at a depth of 2 feet. The water barometer stands at 34 feet.

The pressure of the air in the tube

= *pressure at surface of the water in the tube.*

= pressure of the atmosphere + pressure due to depth of 2 feet in water.

= pressure due to depth of 36 feet in water.

If the tube is of uniform bore, the volumes are proportional to the lengths occupied by the air.

Length of tube occupied by air : length of tube :: 34 : 36.

$\therefore$ length of tube occupied by the air $= \frac{34}{36} \times 30$ inches;

$= 28\frac{1}{3}$ inches.

Hence the water rises $(30 - 28\frac{1}{3})$ inches $= 1\frac{2}{3}$ inches.

Ex. 3.—A diving bell is sunk to a depth a feet; the length of the bell is b feet. Find the length of the bell occupied by the air. The height of the water barometer is h feet.

Let x feet be the length CA occupied by the air.

Let a feet be the depth of C below the surface.

If w is the weight of a cubic foot of water, Π the pressure per square foot of the atmosphere, Π' the pressure of the air in the bell, then

$$\Pi = wh;$$

$$\Pi' = \text{pressure at } A;$$

$$= \Pi + w \times \text{depth of } A;$$

$$= wh + w(x + a);$$

$$= w(h + x + a).$$

By Boyle's Law $\frac{\Pi}{\Pi'} = \frac{\text{volume } CA}{\text{volume of bell}} = \frac{x}{b}$.

$$\therefore \frac{h}{h + x + a} = \frac{x}{b}.$$

$$\therefore x^2 + x(a + h) - hb = 0.$$

A quadratic equation to determine x.

EXERCISES.

1. How deep in mercury must a diving bell be sunk so that the mercury may rise and fill one-tenth of the bell? The barometer stands at 29·5 inches.

2. A diving bell is 8 feet long and 5 feet wide. It is sunk in water to such a depth that the water rises 3 feet into the bell. Find the depth it is sunk if the barometer stands at 30 inches.

3. The pressure of the atmosphere is at standard pressure. A diving bell is sunk in water until one-tenth of it is filled with water. What is the specific gravity of the air inside it? A piece of wood (100 cc.) floats half immersed in water outside the bell. How much will be immersed if it is placed inside the bell?

4. The depth of the surface of water inside a bell is found to be 60 feet. The length of the bell is 5 feet. If the water barometer stands at 33 feet, what height has the water risen in the bell? (The bell is cylindrical.)

5. A barometer tube is depressed with its open end downwards into a cistern of mercury until 10 inches of it is above the surface of the mercury in the cistern. If the length of the tube is 33 inches, and the height of the barometer is 30 inches, how high will the mercury rise in the tube?

6. *A diving bell has a volume of 2000 cubic feet, and it is sunk until the bottom is 30 feet below the surface. How much air at atmospheric pressure must be forced into the bell to keep the water at the bottom of the bell?* (Height of water barometer 33 feet.)

(iii) A **Balloon** consists of a flexible envelope, which *contains gas of less specific gravity than air.* The forces acting on a balloon placed in the air are (1) the weight of the envelope and gas downwards, (2) the force of the air upwards, which is equal to the weight of air displaced by the balloon. The condition that a balloon should rise is that the weight of air displaced must be greater than the weight of the balloon. The difference between the weight of air displaced and the weight of the balloon is the *lifting force* on the balloon. As the balloon ascends the pressure of the air becomes less, and therefore the envelope expands and displaces more air. By Boyle's Law the density of the gas is proportional to the pressure, and the volume is inversely as the pressure, so that for example, if the pressure be reduced to half, the density is reduced to half, but the volume is doubled, and therefore the mass of air displaced is unaltered. Neglecting, therefore, the change in the weight due to height above the surface of the earth, and the change in the mass of air displaced by the envelope, the weight of air displaced by the balloon is constant so long as the balloon is capable of expanding. The upward force on a balloon is therefore constant until the balloon is fully expanded, and then, as it rises, the weight of air displaced decreases, and therefore the

upward force decreases until it vanishes, and then the balloon comes to rest.

Ex.—The envelope of a balloon when fully inflated has a volume of 100 cubic feet, and its weight is 10 ozwt.; 5·4 oz. of hydrogen (sp. gr. = ·00009) is put into it. What will be the lifting force on the balloon during the time of inflating, and what will be the pressure of the air at the place where it comes to rest? Specific gravity of air at beginning of ascent is ·0013.

The lifting force = weight of air displaced − weight of balloon.

Weight of hydrogen = 5·4 ozwt.

Now, air is $\frac{\cdot 0013}{\cdot 00009}\left(=\frac{130}{9}\right)$ times heavier than hydrogen.

$\therefore$ weight of equal volume of air $= \frac{130}{9} \times 5\cdot 4$ ozwt.;
$= 78$ ozwt.

$\therefore$ weight of air displaced by hydrogen = 78 ozwt.;
weight of balloon = 5·4 ozwt. + 10 ozwt. = 15·4 ozwt.
Lifting force = 78 ozwt. − 15·4 ozwt. = 62·6 ozwt.

To find the pressure of the air when the balloon is at rest.

When the balloon is at rest,

weight of air displaced by balloon = weight of balloon;
= 15·4 ozwt.

If s = sp. gr. of the air at the place of rest, then, since volume of inflated envelope is 100 cubic feet,

$$100 \times s \times 1000 \text{ ozwt.} = 15\cdot 4 \text{ ozwt.}$$

$$\therefore s = \frac{15\cdot 4}{100000}.$$

If Π = pressure of the air at beginning of ascent, Π' = pressure of the air at place of rest, then

$$\frac{\Pi'}{\Pi} = \frac{s}{\cdot 0013} = \frac{15\cdot 4}{100000} \div \frac{13}{10000}.$$

$$\therefore \frac{\Pi'}{\Pi} = \frac{15\cdot 4}{130}; \quad \Pi' = \frac{15\cdot 4}{130} \times \Pi.$$

If Π = 15 lbwt. per square inch, Π' = 1·777 lbwt. per square inch *nearly*.

EXERCISE.

A balloon has a volume of 500 cubic feet when fully inflated. Find the weight of air it will displace if the specific gravity of air is ·0013. If it is filled with gas of one quarter the specific gravity of air, what will be the upward force on the balloon, neglecting the weight of the balloon?

6. **Gauges.** It is convenient in this place to enumerate the different forms of gauges for measuring the pressure of fluids, whether liquids or gases.

(i) Gauges for measuring pressures which are lower *than the pressure of the atmosphere are frequently called* vacuum gauges. A simple form of vacuum gauge, *the cistern vacuum gauge,* is constructed by taking a glass tube and dipping one end into a vessel of mercury, and connecting the other end to the fluid, the pressure of which is to be measured. The atmospheric pressure will drive the mercury up the tube, and the difference between its height and the height of the barometer is a measure of the pressure. This form of gauge is largely used to measure the "vacuum" of the condenser of a steam engine.

Another form of the same gauge, *the open tube vacuum gauge,* is the one described on page 127, where the tube is formed into a U tube, and the difference of level of the mercury in the two branches gives the *amount the pressure of the atmosphere exceeds the* pressure which is measured.

Both these gauges require the pressure of the atmosphere to be measured. A gauge, however, can easily

be formed on the same principle as the siphon barometer. A U tube, such as that on page 129, is filled with mercury so that there is a vacuum at the closed end; the open end is connected to the vessel, the pressure in which is to be measured. In this case the difference of level in the two branches is a direct measure of the pressure of the fluid; just as when it is open to the atmosphere, the difference of level is a measure of the atmospheric pressure. This, *the closed tube vacuum gauge*, or **Siphon Gauge**, is the form of gauge used in air pumps, and it is obvious that if only very low pressures are to be measured, the length of the tube need not be very long.

(ii) Of gauges for measuring pressures above atmospheric pressure, a simple form has been described in Exp. I., page 65. If only small pressures are to be measured, water may be used. But if great pressures are to be measured, mercury must be used, and the length of one arm of the U tube must be sufficiently long. In order to measure very large pressures up to 10 atmospheres, *i.e.* about 300 inches of mercury, the length of the tube becomes very inconvenient, so that this form of gauge is only used to check the graduations of other more compact forms. Such a compact form can be made by closing the end of the tube, and thus allowing the mercury to be driven up against the enclosed air. The pressure of the enclosed air can be estimated from Boyle's Law. For example, suppose 12 inches of air is enclosed at atmospheric pressure (30 inches of mercury), then, if this

volume is reduced to half, the pressure of the air is 60 inches of mercury, and the increase in the pressure of the enclosed air is equivalent to 30 inches of mercury; so that a rise of 6 inches in the closed tube is equivalent to a rise of 30 inches when it was open. If the mercury rises another 3 inches, the pressure of the enclosed air is again doubled, *i.e.* becomes equivalent to 120 inches of mercury. Hence such a gauge can be graduated at a given temperature, and a table of corrections can be applied for other temperatures.

(iii) The gauges which depend upon the use of a liquid are often very inconvenient, both on account of their size and liability to accident. More convenient gauges are constructed by making use of the elastic property of solids. The chief forms of metallic gauges are:

(*a*) The **Aneroid** barometer. This is used to measure the pressure of the atmosphere, and consists of a metallic vessel with an elastic cover. This vessel has the air taken from it, and the elastic cover is acted upon by the atmospheric pressure. Any change in the pressure of the atmosphere produces a different deflection of this membrane, and the motion of the membrane is communicated to a multiplying system, and is magnified by a pointer moving over a dial. The dial is graduated by comparison with a standard mercury gauge.

(*b*) **Bourdon's Gauge.** This consists of a hollow conical steel tube bent into the form of a crescent.

The vertex is closed, and the other end is attached to the vessel, the pressure in which is to be measured. The effect of increase of pressure is to straighten the tube, so that as the pressure increases, the tube straightens, and its end moves over an arc. The motion of the vertex is communicated to a pointer moving over a dial, and the dial is graduated by comparison with the standard mercury gauge.

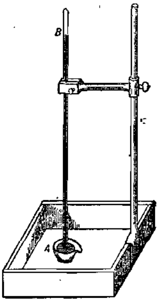
(*c*) **Watt's Indicator.** This is a valuable form of *gauge used to determine the pressure of fluid when it is* continually changing. The essential part of the gauge consists of a metallic piston, which is accurately made so as to move almost gas-tight in a cylinder. The motion of the piston is resisted by a strong steel spiral spring, through the centre of which the piston rod passes. When, therefore, the force of the fluid is exerted on the piston, the spring is compressed, and the amount of compression is indicated by a multiplying lever. The pointer at the end of the lever makes a mark on paper which is fixed on a drum. If the drum is kept in motion, the pressure of the fluid at any moment can be found, as the pencil will trace out a curve on the paper. The indicator is of the greatest possible value in finding the pressure of the steam in the cylinder of a steam engine at any point of the stroke. A detailed account of it will be found in books on Heat or on the Steam Engine.

EXPERIMENTS ON CHAPTER V.

EXP. I.—*To construct a mercury barometer, and find the height of the barometer.*

Read pages 129, 130, and Note 4, page 133.

The tube should be from 32 to 36 inches long, and of about ·5 to ·7 cm. bore. To fill it, it is best to use a small funnel, and if the mercury is not quite clean, make a paper funnel to fit the glass one, and make a few holes in the paper by means of a needle. In this way the mercury is freed from dust. Fill the tube to about $\frac{1}{4}$ inch from the top, and then close the end firmly by the finger. Now incline the tube so that the large air bubble ascends to the top, then make the bubble run the whole length of the tube, gathering together all the smaller air bubbles. Now fill up the tube to the brim, and close it with the finger so as to exclude all the air from the tube. Now invert the tube, and place the closed end into mercury in the cup. See that there is no air near the mouth of the tube, and that the finger is well below the mercury. Now withdraw the finger, and the mercury in the tube will fall to a height depending upon the pressure of the atmosphere.

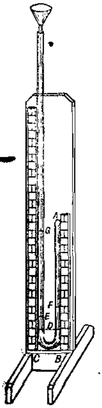


The tube should be put in a vertical position. This can be done by arranging it so that it is in the same straight line with its image formed by reflection at the

surface of the mercury in the vessel. Now take a metre scale and measure the height of the mercury in the tube above the *surface* of the mercury in the basin. This will give the height of the barometer.

EXP. II.—*To prove Boyle's Law.*

Take a piece of glass tubing of narrow ($\frac{1}{4}''$) bore, as uniform as possible and about 150 cm. long. Bend it into the form of a U tube with one branch about 100 cm. long, the other about 30 cm. Close the end of the short branch. Fix this tube on a stand (as in Exp. I. Chap. III.). Draw a horizontal line *CB* on the board. Pour mercury into the tube until it fills rather more than the curved part of the tube. Now measure the height *AB* of the top of the short arm above *CB*. Measure also the heights of the surfaces *D* and *E* above *BC*.



(i) The difference between *AB* and *BD* is the length of the tube occupied by the enclosed mass of air.

(ii) The difference between *CE* and *BD* gives the difference of pressures in mm. of mercury of the air in *BA* and in the open branch.

Read the height of the barometer in mm. of mercury. Then the pressure of the air in *AD* can be found.

Example.—The height *AB* (taken as near as possible to the inside of the tube at *A*) was found to be 385 mm.; *CE* was 33 mm.; *BD* was 18·5 mm.; and the height of the barometer was 764·9 mm.

(i) Therefore the length of tube occupied by the mass of enclosed

(ii) The difference of pressure between the air in AB and that in the open branch $= 33 - 18{\cdot}5$, *i.e.* $14{\cdot}5$ mm.

Hence the pressure of the enclosed mass of air $= 764{\cdot}9 + 14{\cdot}5$, *i.e.* $779{\cdot}4$ mm. of mercury.

Now pour in more mercury to increase the height in the long branch by a few centimetres (say 6 to 12 *cm.*), and measure the heights of the new surfaces F and G above BC. Find, as in the first case, (i) the length of tube occupied by the enclosed mass of air; (ii) the pressure of this mass of air in mm. of mercury.

Example.—The height of BF was found to be $36{\cdot}5$ mm.; CG was $92{\cdot}5$ mm. Hence

(i) Length of tube now occupied by the same mass of air
$= 385 - 36{\cdot}5$, *i.e.* $348{\cdot}5$ mm.

(ii) The difference of pressures $= 92{\cdot}5 - 36{\cdot}5$, *i.e.* $56{\cdot}0$ mm., and therefore the pressure of the enclosed mass of air $= 764{\cdot}9 + 56{\cdot}0$, *i.e.* $820{\cdot}9$ mm. of mercury.

To test the accuracy of Boyle's Law.

By the above experiment the length of tube occupied by the air is known, and the pressure of the air is known. Hence, when the air is compressed by increase of pressure, the new length of tube occupied by the air can be found by calculation if Boyle's Law is true, and thus it can be compared with the length actually found by experiment.

Example.—The air occupies $366{\cdot}5$ mm. of the tube when its pressure is $779{\cdot}4$ mm. of mercury. What length of tube will it occupy if the pressure is made $820{\cdot}9$ mm. of mercury?

When the pressure is $779{\cdot}4$ mm. the air occupies $366{\cdot}5$ mm. of tube.

If the pressure is 1 mm. the air would occupy $366{\cdot}5 \times 779{\cdot}4$ mm. of tube.

∴ when the pressure is $820{\cdot}9$ mm. the air occupies

$$\frac{366{\cdot}5 \times 779{\cdot}4}{820{\cdot}9} \text{ mm. of tube.}$$

∴ the air occupies $347{\cdot}9$ mm. of the tube.

If therefore Boyle's Law is true, the air ought to occupy 347·9 mm. of the tube under the new pressure; but the experiment has shown it to occupy 348·5 mm. of the tube, a difference of only ·6 mm.

The student should make at least six such measurements. This can easily be done by pouring in additional mercury to increase the height in the longer branch by about 10 to 15 cm. each time.

The results of the measurements and of the calculations can be entered in a table.

Height of barometer = 764·9 mm.
Height of top of closed arm = 385 mm.

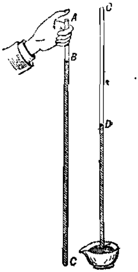
	Height of mercury in long arm in mm.	Height of mercury in short arm in mm.	Pressure of the enclosed air in mm. of mercury.	Volume of the enclosed air in mm. of the tube.	Volume of the enclosed air by calculation.	Difference in mm. of the tube.
1	33·0	18·5	779·4	366·5		
2	92·5	36·5	820·9	348·5	347·9	·6
3	247·0	79·0	932·9	306·0	306·2	− ·2
4	387·0	111·5	1040·4	273·5	274·5	−1.0
5	562·0	144·5	1182·4	240·5	241·6	−1·1
6	835·0	185·0	1414·9	200·0	201·8	−1·8

NOTE.—The difference between the theoretical volume and the observed volume is in no case considerable, but it will be observed that the volume (except in second case) is always less than what it would be according to Boyle's Law. This is due to the fact that the gas experimented upon is not simply air, but a mixture of air and a small quantity of vapour of water. Now, if the volume of a vapour be diminished, the pressure does not increase so rapidly as in the case of a perfect gas; and in the extreme case, when the vapour begins *to condense, the pressure does not increase at all. Hence, on increasing the pressure* of the gas in the tube, the vapour does not exert so great a resistance to change of volume as it would do if it were a perfect gas, and hence the volume is less than it would be if there were no vapour; and as the pressure of the vapour increases less and less rapidly as the volume decreases, the observed volume becomes less and less than the calculated

volume. To test the truth of Boyle's Law with very great accuracy, great care is taken to dry the air, and to ensure that the mercury is perfectly dry. If the student breathes on the tube so as to slightly warm the enclosed air, it will be found to expand, and thus at a greater volume exert a greater pressure. It must be remembered that Boyle's Law is only true when the gas is kept at the same temperature, and therefore care must be taken to secure uniform temperature.

EXP. III.—*To prove Boyle's Law for pressures less than the pressure of the atmosphere.*

Take a barometer tube and pour clean dry mercury into it to within 10 or 20 cm. of the top. Measure the length *AB* of the tube occupied by air. Now close the tube with the finger. This will enclose a volume *AB* of air at the pressure of the atmosphere. Invert the closed end into a basin of mercury, just as if to form a barometer. The air will rise to the top, and so long as the end *E* is closed by the finger it will occupy a length of the tube equal to *AB*, and its pressure will be equal to that of the atmosphere. Now open the end *E*; the enclosed air will expand and exert less pressure. To find the new pressure, measure the height *DE* of the mercury in the tube, and read the height of the barometer; the difference will give the pressure of the air in *CD*. Measure *CD*, the height of the tube now occupied by the enclosed air. Since the length of the tube occupied by the air is known when it is at the pressure of the atmosphere, the length of the tube occupied by the air when at the new pressure can be calculated on the assumption that Boyle's Law is true, and this length can be compared with the actual observed length *CD*.



Example.

Length $AB = 104{\cdot}5$ mm.; height of barometer $= 754{\cdot}5$ mm.

Length $CD = 316{\cdot}8$ mm.; height of $DE = 504{\cdot}0$ mm.

$\therefore$ pressure of air in $CD = 754{\cdot}5 - 504{\cdot}0$ mm., *i.e.* $250{\cdot}5$ mm. of mercury.

When the pressure is 754·5 mm. the volume is 104·5 mm. of the tube.

When the pressure is 1 mm. the volume is $104{\cdot}5 \times 754{\cdot}5$ mm. of the tube.

When the pressure is 250·5 mm. the volume is $\frac{104{\cdot}5 \times 754{\cdot}5}{250{\cdot}5}$ mm. of the tube, *i.e.* the length of CD by theory ought to be 314·7 mm.; but experiment has shown it to be 316·8 mm., a difference of 2·1 mm.

EXAMPLES ON CHAP. V.

1. 200 cc. of cork (sp. gr. ·25) is weighed in air (sp. gr. ·0013) and then in vacuum. Find the difference between the two weights.

2. If a piece of iron weighs 27 gm. in air, what will it weigh in a vacuum? If a piece of cork weighs 27 gm. in air, what will it weigh in a vacuum? (Sp. gr. of air = ·0013.)

3. What is the weight in air of a brass 200-gm. "weight"?

4. A mercury barometer tube is found to have 2 cm. of water on it. If the height of the mercury is 760 mm., and the pressure of the vapour of the water is 12·7 mm. of mercury, what is the true height of the barometer?

5. The height of the mercury barometer is found to be 29 inches. A small quantity of air is admitted into the tube, and the height is found to be 20 inches. Find the pressure of the air inside in lbwt. per sq. inch.

6. The mercury barometer is found to fall one inch. Find the corresponding fall in a water barometer.

7. If the pressure of the atmosphere is 13 lbwt. per sq. inch, what is the height of the mercury barometer?

8. In a siphon (or U tube) barometer the mercury is found to rise ·5 cm. What is the increase of pressure in mm. of mercury, and in dynes per sq. cm.?

9. The height of the barometer at a particular place is 500 mm. of mercury. Find the pressure in dynes per sq. cm., the weight of a gramme at the place being 980 dynes.

10. Find the volumes of the following masses of gas:

(*a*) 200 cc. at 760 mm. of mercury pressure when pressure becomes 200 mm. of mercury.

(*b*) 650 cc. at 15 mm. of mercury pressure when pressure becomes 1000 mm. of mercury.

(*c*) 60 cubic inches at 14 lbwt. per sq. inch when the pressure is reduced to 5 lbwt. per sq. inch.

11. A closed cylinder 1 foot 6 inches long is fitted with an air-tight piston. When the piston is $1\frac{1}{2}$ inch from one end it is filled with air at a pressure of 15 lbwt. per sq. inch. Find the pressures in the two parts of the cylinder when the piston is moved (*a*) 1 inch, (*b*) 2 inches, (*c*) 3 inches towards the more distant end of the cylinder.

12. A cylinder closed at one end is fitted with an air-tight piston. Air is put in the closed vessel thus formed, and in order to measure its pressure it is connected with the top of a long tube dipping in a basin of mercury. When the length of the closed cylinder is 1 inch, the mercury in the tube is at the same level as that in the basin. Find the respective heights the mercury will rise when the piston is drawn out 1 inch, 2 inches, $2\frac{1}{2}$ inches, $3\frac{1}{4}$ inches, the height of the mercury barometer being 760 mm.

13. A barometer tube before being inverted is filled with mercury to half-an-inch from the top. After inversion the air fills 10 inches of the tube, and the height of the mercury is 29 inches. What is the true height of the barometer?

14. A bubble of air is allowed to enter a barometer tube. It depresses the mercury ·1 inch, and occupies a volume of 3 cubic inches. What was its volume at the pressure of the atmosphere, the height of barometer being 30 inches?

15. A barometer stands at 30 inches, and the space occupied by the Toricellian vacuum is then 2 inches. If now a bubble of air, which would at atmospheric pressure occupy half an inch of the tube, be introduced into the tube, prove that the surface of the mercury in the tube will be lowered 3 inches. If this false barometer stands at 28 inches, show that the true height of the barometer is 31·75 inches.

16. A barometer stands at 29·5 inches, and the space occupied by the Toricellian vacuum is 3 inches. A bubble of air is seen to rise into the vacuum and depress the column by 2·5 inches. What volume would the bubble occupy at atmospheric pressure?

17. What is the height of the mercury barometer when the pressure of the atmosphere is a million dynes per sq. cm.?

18. What is the height of the water barometer when the pressure of the atmosphere is 10 lbwt. per sq. inch?

19. A vessel containing 20 gm. of oxygen at 20 lbwt. per sq. inch pressure is put in communication with a vessel containing 12 gm. of hydrogen at 5 lbwt. per sq. inch pressure. Find the pressure in each vessel after equilibrium of pressure is restored.

20. Find the pressure which 10 gm. of air placed in a vessel of 10 litre capacity will exert. (The temperature 0°C.)

21. The pressure of a mass of air at 0° C. is found to be 795 mm. of mercury, and the volume is 9·95 cc. Find the mass of the air.

22. Find the volume of 15 gm. of nitrogen at 0° C. and 800 mm. of mercury pressure.

23. The barometer stands at 30 inches; an open tube vacuum gauge stands at 25 inches. Find the pressure in lbwt. per sq. inch.

24. The surfaces of the mercury in a closed tube vacuum gauge stand respectively at 5 inches and 12 inches above the same horizontal line. Find the pressure in inches of

25. An open tube mercury gauge is used to measur the pressure of a vapour which is known to be 120 mm. c mercury. The barometer stands at 756 mm. of mercury Find the height of the mercury in the gauge.

26. A closed tube vacuum gauge is used to measur the pressure of a gas in a large vessel. The difference c level is 10 cm. If half the gas is taken out of the vesse find the difference of level.

27. The height of the barometer is 30 inches; a pres sure gauge is used to determine the pressure of steam in boiler. The height in the long arm is 80 inches, and i the short arm 10 inches. Find the pressure of the stean in lbwt. per sq. inch.

28. A closed tube pressure gauge is made of uniforn glass tubing. When the pressure is due to 30 inches c mercury, the mercury in the two branches is at the sam level, and the length of the tube occupied by the enclose air is 20 inches. Find the length of the tube occupied b the air when the pressure is made respectively 45, 60, 75 90, 95, 100 inches of mercury. If the tube were graduate into equal divisions, would each division correspond t equal change of pressure?

29. An open canister, $4\frac{1}{32}$ inches high, is inverted ove a vessel of mercury, and pressed down until its bottom is i the surface of the fluid. Find how high the mercury wil have risen within the canister, taking 30 inches to be th height at which the mercury barometer stands.

30. A bubble of gas has a volume of $\frac{1}{2}$ cubic inch whe released under water at a depth of 100 feet below th surface. Find its volume on reaching the surface, th height of the water barometer being $33\frac{1}{3}$ feet.

31. A cylindrical diving bell, 6 feet in height and 4 fee in diameter, is sunk in water until its top is 66 feet belov the surface. The height of the water barometer is 34 feet Find the height to which the water will rise inside th

32. A body is floating on the surface of the water inside a diving bell. How will the body behave as the diving bell is sunk?

33. A rigid body just floats totally immersed near the surface of water; it is depressed to a considerable depth and then let go. What happens?

An elastic envelope containing air just floats totally immersed near the surface of deep water; it is depressed to a considerable depth. What will happen?

34. A conical diving bell is sunk in water until the water rises half way up the bell. If the water barometer stands at 34 feet, how deep is the bell sunk?

35. If the mercury barometer stands at 28 inches, what is the pressure of the atmosphere per square inch, and what will be the corresponding height of the glycerine barometer? (Sp. gr. of mercury = 13·5, of glycerine 1·25.)

36. A mass of 5 cubic feet of air at pressure of 15 lbwt. on the square inch is forced into a vessel of 2 cubic feet. What is the new pressure?

37. A mass of 3 cubic feet of air at pressure of 15 lbwt. per square inch is placed with 3 cubic inches of air at pressure of 28 lbwt. per square inch into a vessel of 2 cubic feet. What will be the pressure of the mixture?

38. If the specific gravity of air at pressure of $14\frac{3}{4}$ lbwt. per square inch is ·0013, what will be the weight of 5 cubic feet of air at pressure of 18 lbwt. per square inch? What will be the volume of 5 oz. of air at the same pressure?

39. A mass of 3 oz. of air at 12 lbwt. per sq. inch pressure is placed in a vessel of 20 cubic feet. What will be the pressure on the vessel?

40. A vessel contains 30 cubic feet of air at atmospheric pressure. What volume of air at atmospheric pressure must be forced into the vessel so as to make the pressure 30 lbwt. per square inch? The pressure of the atmosphere is $14\frac{3}{4}$ lbwt. per square inch.

41. What volume of air at atmospheric pressure must be introduced into the Toricellian vacuum to depress the column of mercury 5 inches, the area of section of tube being 1 inch? The barometer originally stood at 30 inches.

42. The area of the section of a Boyle's tube is 1 square inch. Mercury is poured into it and adjusted so as to stand at the same level in the arms, the volume of air in the short arm being 5 cubic inches. How much mercury will have to be poured into the longer branch to make the difference of level 6 inches?

43. What volume of air at atmospheric pressure will have to be removed from the short arm to reduce the surfaces to the same level.

44. If a body be suspended by an elastic string, explain how the length of the string will be affected by a fall in the barometer.

45. If the longer branch of a siphon be inserted into a closed vessel from which the air has been partially exhausted, describe the effects on the action of the instrument.

46. Over what height can water be carried by a siphon when the barometer stands at 29·5 inches, the specific gravity of mercury being 13·5?

47. If the reading of a glycerine barometer be 320 inches when that of a mercurial barometer is 29·755 inches, find the specific gravity of glycerine, that of mercury being 13·5.

48. State clearly the conditions for the ascent of a balloon, and show what would be the effect of a variation of barometric pressure on the lifting power of a balloon.

Taking 7·67 lbwt. as weight of 100 cubic feet of air, find approximately the volume of hydrogen (sp. gr. = ·07 times that of air) which a balloon must contain in order to have a lifting power of 713 lbwt.

49. Explain the action of a siphon.

Water is flowing out of a vessel through a siphon. What would take place if the pressure of the atmosphere were

removed and afterwards restored (1) when the lower end is immersed in water, (2) when it is not?

50. A diving bell is stationary at a certain depth under water when a block of wood falls into the water off a shelf inside the bell. Show that the water will rise within the bell and that, notwithstanding, the bell will contain less water than it did before.

51. A hollow cylinder, closed at one end, is placed with its open end on the surface of a fluid, and forced vertically downwards until the closed end of the cylinder is on the surface of the fluid. Show that if the height of the cylinder be $6h$, the air in the cylinder will occupy one-third of its volume, h being the height of the fluid barometer.

52. A diving bell is lowered first in water, and afterwards to the same depth as before in a fluid of less specific gravity than water. Does the water or the other fluid rise higher in the bell? In which case is the pull of the chain greater? Give your reasons in each case.

A diving bell is lowered in a lake until two-thirds of it is filled with water. Show that if d be the depth of the top of the bell below the surface, the height of the bell is $3(2h-d)$ where h is the height of water barometer.

53. A cylindrical test-tube is held vertically and immersed downwards in water. When the middle of the test-tube is 32·75 feet below the surface of the water in the vessel, the water fills half of the tube. Find the atmospheric pressure in pounds-weight per square inch.

54. An air bubble at the bottom of a pond 10 feet deep has a volume of ·00006 cubic inch. Find what its volume will become when it just reaches the surface. The barometer stands at 30 inches.

55. A tumbler-full of air is placed mouth downwards under water at such a depth that the surface of the water inside it is at a depth of $25\frac{1}{2}$ feet. Compare the weight of a cubic inch of air in the tumbler with that of a cubic inch of air outside, the barometer standing at 30 inches.

56. A piston works air-tight in a cylinder which is open above, and of which the internal diameter is 3 feet. If the pressure of the atmosphere upon the upper surface of the piston is 15 lbwt. on a square inch, and if the space below the piston be filled with air exerting an equal pressure, what weight will be required to compress the air inside to two-thirds of its original volume?

57. When the reading of a mercury barometer is 760 mm., what will be the readings of a water and of a glycerine barometer, the specific gravity of mercury being 13·6, and of glycerine 1·26?

58. A litre of air weighs 1·29 gms.; a litre of coal gas weighs ·52 gm. A balloon contains 4,000,000 litres of coal gas, and the weight of the envelope and appendages is 1,500,000 gmswt. What additional weight will it be able to sustain in the air?

59. Two quarts of a gas at pressure of 30 lbwt. per square inch and three quarts at pressure of 20 lbwt. per sq. inch are forced into a quart vessel. What will be the pressure?

60. A cubic foot of air weighs 1·2 oz. A balloon contains 1·5 cubic feet of coal gas, and the envelope of the balloon weighs 1 oz. The balloon floats in the middle of a room. Find the specific gravity of the coal gas.

61. A litre of air at 0° C., and under 760 mm. barometric pressure, weighs 1·293 gms. Show how to find the weight of 73 litres at same temperature and under 1000 mm. pressure.

62. The mercury barometer stands at 30 inches; the section of the tube is 1 square inch. The vacuum above the mercury in the tube is 30 cubic inches. As much air is passed up the tube as depresses the mercury to 29 inches. What would be the volume of the air at atmospheric pressure?

63. A cylindrical diving bell 10 feet high is sunk to a certain depth, and the water is observed to have risen 2 feet

in the bell. As much air is then pumped in as would fill $\frac{87}{410}$ of the bell at atmospheric pressure, and the surface of the water in the bell is noticed to sink through 1 foot. Find the depth of the top of the bell, and the height of the water barometer.

64. What is the law connecting the pressure of a gas with its density when the temperature remains constant? Describe an experiment by means of which this law can be verified.

If a vessel containing 20 grs. of oxygen at a pressure of 10 lbwt. per square inch be put into communication with another containing 5 grs. of hydrogen at a pressure of 30 lbwt. per square inch, what will be the final pressure in either vessel? The density of oxygen is 16 times that of hydrogen when the pressures are equal.

65. A cylindrical diving bell of length $\frac{h}{4}$ is sunk in water till its lowest part is nh below the surface. If the water fill one-fifth of the bell, show that the bell contains air whose volume at atmospheric pressure would be $\frac{4}{5}(n + \frac{19}{20})V$, V being the volume of the bell and h the height of the water barometer.

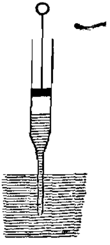
66. A balloon capable of containing 400 cubic feet of a gas is partially filled with 104 oz. of coal gas, the specific gravity of which is ·4 times that of air at atmospheric pressure. Find the lifting force on the balloon, the density of the air at the place where it comes to rest, and the height above the sea of the position of rest. The specific gravity of air at sea level is ·0013, and the weight of the envelope is 2 lbwt.

CHAPTER VI.

On Hydrostatic Machines.

The Hydrostatic Machines may be considered under three classes: (1) the water pumps; (2) the air pumps; (3) the hydraulic presses.

1. The **Water Pump** is a machine for raising water, and its action depends essentially upon the *principle* of **Suction.** This *principle* can be explained by considering the action of an ordinary syringe. A syringe consists of a cylinder, one end of which is worked into a *narrow tube*. The *cylinder is fitted with an air-tight piston capable of moving* in it. If the piston is at the lowest point of the cylinder, and the tube be placed into water, then, on drawing the piston up, it will be found that the water rises into the cylinder. Now, it has been shown in the preceding chapter that if a tube closed at one end be inverted into water,



and the air exhausted from it (this was done in a barometer tube by first filling it with the liquid, and then inverting the tube, when the liquid fell and left a vacuum at the top), the water will rise to a height of 34 feet. *If the air is only partially exhausted the water*

will rise until the pressure of the air together with the pressure due to the height of the water in the tube is equal to the pressure of the atmosphere. If, therefore, the *pressure of the air in the tube is less than the pressure of the atmosphere, the water will be driven up the tube.* On raising the piston of the syringe the mass of air within the tube expands into the cylinder, and the pressure is diminished, and becomes less than the pressure of the atmosphere, and the pressure of the atmosphere drives the water into the cylinder. In ordinary language, the water is said to rise by *suction.*

The principle of suction depends essentially upon the atmospheric pressure, and its application consists in removing, or partially removing, the pressure of the atmosphere from a portion of the surface of a liquid or solid; and therefore, in the case of a liquid, the atmospheric pressure upon the remaining surface of the liquid causes the liquid subjected to the diminished pressure to rise. The effect of suction in the case of solids is to cause two solids, which are pressed closely together so as to exhaust the air from between them, to adhere to each other. Thus, if a disc of leather attached to a string be moistened and pressed closely to a stone, the leather will adhere to the stone, and the stone may be supported by the string.

The action of drinking water by drawing it through a tube is a familiar illustration of the principle of suction. The operator causes the air within the tube to expand into his mouth, and the water consequently rises.

When the piston of a syringe is driven back, the pressure of air is increased, and the water is forced out. If, however, means were adopted for preventing the air in contact with the water from being compressed, the water would remain in the tube, and in the next upward stroke the water would rise higher in the tube. A **Valve** is used for this purpose. It is a trap-door, which opens and allows water or air to pass through an opening in one direction, but closes and prevents the air or water returning in the opposite direction. Valves are of different construction.

The diagrams show four forms of valves in common use in water pumps. The first (Fig. 1) is a flat disc turning about a hinge,

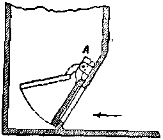


Fig. 1.

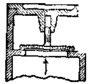


Fig. 2.

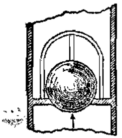


Fig. 3.

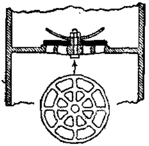


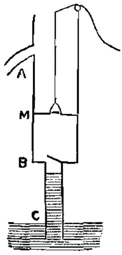
Fig. 4.

opens and closes a circular passage. This is called the hang-p valve. The second (Fig. 2) is a lift valve, and is a metal pable of moving in a vertical direction. It is provided with a stem which moves in a cylindrical socket, and thus guides the

valve's motion. The third (Fig. 3) is the ball clack valve—a brass sphere, which accurately fills a spherical cup, into which a pipe leads. The fourth (Fig. 4) is the butterfly valve—a disc of oiled india rubber, fixed at its centre. The orifice has a grating over it, so that the valve may not be bent through it. The valve opens the orifice by bending about its centre, and to prevent it bending too far a guard is placed over it. In the figure the valve is shown in section, and the grating is shown in plan below it. This form of valve is much used in the exhaust pumps of steam engines. The valves of the air pumps described below are generally made of oiled silk stretched over the orifice.

(i) *To explain the action of the Common Pump.*

The **Common Pump** consists of a cylinder *AB* traversed by a piston *M*, and communicating by means of a pipe with the water to be raised, and having a spout at *A*.



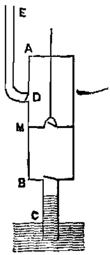
In the piston and at the bottom of the cylinder are valves opening upwards.

Suppose the pipe *BC* to be full of air, and the piston at *B*. When the piston is raised the air in *BM* becomes rarer (and therefore at less pressure) than the air in *BC*, the valve at *B* is opened, and part of the air in *BC* rushes into *BM*. The pressure in *MC* is now less than the pressure of the atmosphere, and the water is driven up the pipe. As the piston descends, the air in *BM* becomes denser than the air in *BC* (and therefore at greater pressure), the valve at *B* is closed, and the air

in BM becoming denser than the air outside, the valve at M is opened, and the air rushes out. In the next stroke more air is driven out, and the water is driven by the pressure of the atmosphere higher up the pipe, until after a few strokes the water passes into BM, and through the valve at M, and is lifted to the spout at A.

(ii) *To explain the action of the Lift Pump.*

The **Lift Pump** consists of a cylinder AB closed at the top, and traversed by a piston M (the piston rod working through an air-tight collar in the top of the cylinder) and communicating with the water by means of a pipe. There is also a tube DE leading upwards from the top of the cylinder. In the piston, at the bottom of the cylinder, and at D are valves opening upwards.

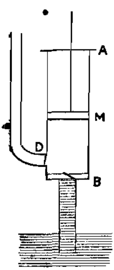


Suppose the pipe to be full of air, and the piston at B. When the piston is raised the air in BM becomes rarer (and therefore at less pressure) than the air in BC, the valve at B is opened, and part of the air in BC rushes into BM. The pressure in MC is now less than the pressure of the atmosphere, and the water is driven up the pipe. As the piston descends the air in BM becomes denser than the air in BC, the valve at B is closed, and the air in BM becoming denser than the air in AM, the valve at M is opened, and air rushes into AM. As the piston ascends the air in MA becomes denser than the

air outside, opens the valve at *D*, and part of it is forced out. After a few strokes the water rises and passes through the valve at *M*, and then through the valve at *D*. At each stroke upwards more water is lifted through *D* and up the tube.

(iii) *To explain the action of the Force Pump.*

The **Force Pump** consists of a cylinder *AB* traversed by a solid piston *M*, and communicating by means of a pipe with the water to be raised. There is also a tube leading upwards from the bottom of the cylinder. At *D* and *B* are valves opening upwards.



Suppose the pipe to be full of air, and the piston at *B*. When the piston is raised the air in *BM* becomes rarer (and therefore at less pressure) than the air in the pipe, the valve at *B* is opened, and the air in the pipe rushes into *BM*. The pressure in *MC* is now less than the pressure of the atmosphere, and the water is driven up the pipe. As the piston descends the air in *BM* becomes denser (and therefore at greater pressure) than the air in the pipe, the valve at *B* is closed, and the air in *BM* becoming denser than the air outside, the valve at *D* is opened, and the air rushes out through the tube. In the next stroke more air is driven out, and the water is driven by the pressure of the atmosphere higher in the pipe, until after a few strokes the water passes into *BM*

and through the valve at D. During the upward part of the stroke the valve at D is closed, and the water prevented from falling in the tube, and each downward stroke drives the water higher up the tube.

The height to which the water can be lifted in the pipe, *i.e.* the height to which the atmosphere will drive the water up the pipe, depends upon the nature of the "vacuum" which can be formed below the piston. If a perfect vacuum could be formed, then the greatest height to which the water would rise is the height of the water barometer, on the average 34 feet. It is impossible to obtain a perfect vacuum, and it is found practically that the greatest height to which water will rise in the pipe of a good pump is about 25 feet. The vacuum is limited by several causes: (i) The valves have weight, and if they are placed so as to move vertically, a force is required below the valve in order to lift it. If the valve is placed so as to move horizontally, then there must be force required to overcome the friction of its supports. If the valve is made of india rubber, then force is required to bend the india rubber. Hence the pressure can never be less than that necessary to provide this force. (*See* Ex. 4, p. 183). (ii) It is practically impossible for the piston to move the whole length of the cylinder, so that when it is at the bottom of its stroke there is a "clearance" space left between it and the end of the cylinder. Hence there will always be some fluid above the suction valve (as the valve in the pipe is called), and force will be required to overcome the force of the fluid on the top of the valve. (iii) The valves may not accurately fit in their seats, and hence, during the down stroke, fluid may leak through the suction valve and increase the pressure in the pipe; and during the up stroke air may leak through the discharge valve. This, in the case of the common pump, can be prevented by pouring water on to the top of the piston, thus making the piston air-tight during the up strokes. This common practice is known as "priming" the pump. (iv) The piston may not fit its cylinder accurately. To make a piston work fluid-tight in a cylinder grooves are formed in its edge and filled with hemp. In the force pump the piston is frequently a large solid piston called a plunger. This is made of metal, and it works air-tight through a collar at the top of the cylinder. This collar (called the "stuffing-box") is filled with hemp, well plaited and greased, and forced tight into its place by a ring of metal (called a "gland"). This is shown in section in the drawings of the force pumps on pages 184, 186, where the dark ring surrounding the piston rod in the figure on page 184 and that surrounding the

plunger in the figure on page 186 represents the hemp, which it will be seen is kept in place by a brass ring (the gland) which can either be screwed or bolted down upon it.

NOTE 1.—*To find the force on the piston in the Common, Lift, and Force Pumps during the upward stroke.*

(i) When the water has not risen into the barrel.

The pressure at any point of the upper surface of the piston is the pressure of the atmosphere. Let the pressure of the atmosphere $= \Pi$.

The pressure at any point on the under surface is the pressure of the air inside. Let this pressure $= \Pi'$.

Then, if h feet be the height the water has risen, and $w =$ weight of cubic foot of water, $\Pi' + wh = \Pi$.

Now, if the area of piston $= S$ square feet,

$$\text{Force on the piston} = \Pi S - \Pi' S;$$
$$= (\Pi - \Pi') S;$$
$$= whS.$$

(ii) When the water has risen above the piston.

Let h feet be the height the water has risen.

Let h' feet be the height of the piston above the surface of the water outside.

Then $(h - h')$ feet is the depth of the water above the piston.

$\therefore$ pressure at the upper surface of the piston $= \Pi + w(h - h')$;

and pressure at the lower surface $= \Pi - wh'$.

Therefore, if the area of piston $= S$ square feet,

$$\text{Force on the piston} = \{\Pi + w(h - h')\} S - (\Pi - wh') S;$$
$$= whS.$$

Hence in both cases **the force on the piston is equal to the weight of a column of water, the section of which is equal to the section of the piston and the height equal to the height the water has been raised.**

NOTE 2.—*To find the force on the piston rod of a pump during the down stroke.*

In the case of the Common and Lift Pumps, there is no force due to the fluid on the rod during the down stroke.

In the Force Pump as the piston descends the valve D opens, and if at a particular point of the stroke h is the height of the water in the pipe above the lower surface of the piston, and the water is being driven uniformly up the pipe into the atmosphere; then the pressure on the piston is $wh + \Pi$ upwards and Π downwards, *i.e.* the effective pressure is wh upwards. If S square feet is the area of the piston, the force on the piston is whS, *i.e.* is equal to the weight of a column of water, the height of which is the height of

the surface of the water in the pipe, and base is equal in area to the area of the piston.

If the water is being driven into a vessel within which the pressure is Π' (greater than the pressure of the atmosphere, as is the case when a force pump is used to supply water to a boiler), then the force on the piston $= (wh + \Pi' - \Pi) S$.

Ex. 1.—If the section of the pipe be 12 square inches, the section of the harrel 1 square foot, and the height of the lower valve above the surface of the water 20 feet, find the length of stroke that the water may just rise into the barrel at the end of the first stroke.

$$\text{The volume of the pipe} = 20 \times \tfrac{12}{144} \text{ cubic feet};$$
$$= \tfrac{5}{3} \text{ cubic feet.}$$

If the length of stroke is x feet, the volume of the barrel $= x$ cubic feet.

Now, if the water just rise into the barrel, the air in the pipe must now occupy x cubic feet of the barrel.

Let Π = pressure of the atmosphere, and w = weight of a cubic foot of water; $\Pi = w \,.\, 33$. And if Π' = pressure of the air in the barrel when the water has just risen into it,

Π' = pressure of atmosphere — pressure due to height of the water;
$= w \,.\, 33 - w \,.\, 20$;
$= w \,.\, 13$.

Now, by Boyle's Law,

$$x : \tfrac{5}{3} :: \Pi : \Pi' :: 33 : 13;$$
$$x = \frac{5 \times 33}{3 \times 13} = 4\tfrac{3}{13} \text{ feet.}$$

Ex. 2.—How high will the water have risen in the pipe in the pump of the preceding example after a stroke of 2 feet?

If y feet be the height the water rises in the pipe, then, if Π' is the pressure of the air in the pump after the stroke,

$$\Pi' = w \,.\, 33 - w \,.\, y.$$

The volume of the air after the stroke = volume of barrel + volume of air in the pipe.

Now, the length of the tube occupied by air after the stroke is $20 - y$ feet.

$$\therefore \text{ volume of air in pipe} = (20 - y) \tfrac{1}{12} \text{ cubic feet.}$$

The volume of air after the stroke

$$= 2 \text{ cubic feet} + (20 - y) \tfrac{1}{12} \text{ cubic feet};$$

$$= \left(2 + \frac{20-y}{12}\right) \text{ cubic feet};$$
$$= \left(\frac{44-y}{12}\right) \text{ cubic feet.}$$

And by Boyle's Law,

$\Pi' : \Pi$ :: first volume of air : second volume of air.

$$\therefore 33 - y : 33 :: \tfrac{5}{3} : \frac{44-y}{12}.$$
$$\therefore (33-y)\left(\frac{44-y}{12}\right) = 33 \times \tfrac{5}{3}.$$
$$\therefore (33-y)(44-y) = 20 \times 33;\ y^2 - 77y + 792 = 0.$$
$$\therefore y = 12{\cdot}23 \text{ nearly, or } 64{\cdot}77 \text{ nearly.}$$

The first value is clearly the one which satisfies the condition of the problem.

Therefore the water rises 12·23 feet.

Ex. 3.—In Ex. 1 find the force on the piston at the end of the first up stroke.

The area of the piston is 1 square foot, and the height the water raised is 20 feet; therefore the force on the piston is equal to the weight of 20 cubic feet of water, *i.e.* equal to $20 \times 62{\cdot}5$ lbwt.

The force on the piston = 1250 lbwt.

Ex. 4.—If the area of a lift valve is 9 square inches, and its weight is $1\frac{1}{2}$ lbwt., find the least pressure which can be obtained below it.

The force on the valve must be at least $1\frac{1}{2}$ lbwt.

∴ the pressure must be such as to produce on 9 square inches a force of 1·5 lbwt.

∴ the least pressure must be 1·5 lbwt. per 9 sq. inches,

i.e. $\frac{1{\cdot}5}{9}$ lbwt. per sq. inch = ·166 lbwt. per sq. inch.

EXERCISES.

1. Water is lifted a height of 22 feet by a common pump, the diameter of the piston being 4 inches. Find the force on the rod during the up stroke.

2. Water is driven to a height of 150 feet by a force pump, the diameter of the plunger being 1·5 feet. Find the force on the pump rod in the down stroke.

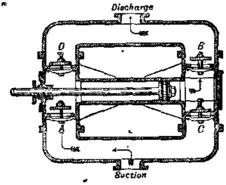
3. If the plunger of a force pump has a cross section of 6 square inches, and works 85 feet below a cistern, find the force which must be exerted along the piston rod in order to drive water into the cistern.

4. A force pump is used to drive water into a boiler, the pressure in which is 80 lbwt. per sq. inch. If the diameter of the plunger is 8 inches, find the force which must be exerted along the rod. To what height would this force drive water?

5. The weight of a lift valve is $\frac{3}{4}$ lbwt., and the diameter of the cross section of the orifice it closes is 2 inches. Find the greatest height to which water can be lifted, the height of water barometer being 34 feet.

NOTE 3.—*Methods of producing a continuous discharge.*

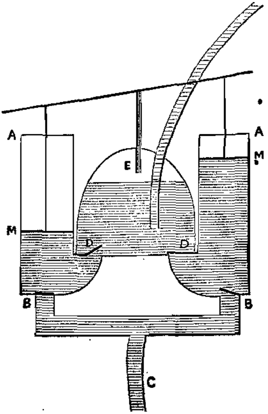
(i) The **Double-acting Force Pump.** In the force pump as above described the flow of water will only take place during the down stroke. The pump is said to be *single acting.* A double-acting force pump is one in which discharge takes place in both the up and down stroke. The figure will show how this may be provided for. The valves at A and C open into the cylinder out of the



suction pipe, and they are called the suction valves. The valves at B and D open out of the cylinder into the discharge pipe; they are called the discharge valves. When the piston is moving forward the pressure in the back of the cylinder (*i.e.* in the part of the cylinder on the left side of the piston) becomes less than the pressure in either the inlet or the discharge pipes. Hence the valve A opens and the valve D closes, and water passes into the cylinder

through *A*. At the same time the pressure in front of the piston becomes greater than that in the discharge or inlet pipe, and therefore the valve at *C* is closed, and that at *B* is opened, and water is driven through *B*. On the back stroke *C* and *D* are opened and *A* and *B* are closed, and water passes through *C*, and the water which is in the back of the cylinder is forced through *D*. Hence water is discharged in both the forward and back stroke.

(ii) The **Fire Engine** is another example of a method for obtaining a continuous discharge of water. It consists of two force pumps placed with their axes parallel to each other, so that their piston rods can be worked from the ends of the same lever which turns about its centre. In this case one piston descends and drives out water, whilst the other ascends and draws the water up.

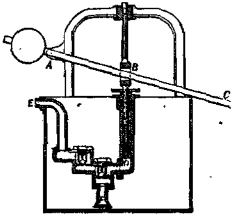


(iii) **The Air-Chamber.** The rate at which the piston is driven will not generally be uniform, but will be greatest somewhere about the middle of the stroke, and zero at the end of the stroke. When the piston is moving at its greatest speed the water will be forced out with great velocity; but when the piston reaches the end of the stroke the velocity of discharge will be much less. Hence the rate at which the water is discharged during a stroke will vary considerably, and the flow of water will be intermittent. To prevent this, and to reduce the straining action on the pump due to the sudden changes in the velocity of the water, a vessel called an *air-chamber* is fitted to all quick-working pumps. This chamber consists of a strong hollow iron cylinder filled with air, and is attached to the discharge pipe, or to a chamber from which the discharge pipe leads, as at E in the figure of the Fire Engine. When the piston is going quickly, part of the force is used to compress the air in the chamber, and thus the velocity of discharge is reduced. When the piston is moving slowly, the air expands and exerts a force on the water, and thus increases the velocity of discharge. In dynamical language, the air in the chamber is receiving energy during the time when the piston is moving quickly, and it gives up this energy when the piston is moving slowly. It performs a function similar to the fly-wheel of an engine.

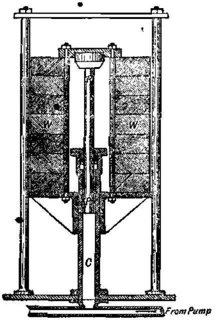
NOTE 4.—*Some uses of the Force Pump.*

The force pump is used for many purposes besides that of raising water.

(i) The figure below is a section of a force pump used in testing the strength of boilers. The discharge pipe E is connected to the boiler, and when the boiler is filled with water a great pressure can



be brought to bear on its sides. This will cause the boiler to enlarge, and the force pump is then worked until the boiler is filled again. In this way (*see* Ex., p. 196) a small force acting at *C* will create a great pressure on the sides of the boiler.



(ii) The force pump is also used to lift the load in an **hydraulic accumulator.** The accompanying figure represents an accumulator. *PP* is a strong metal ram which works water-tight in the cylinder *C*. To the top of the ram is attached a large iron vessel consisting of two concentric cylinders, in which masses of iron are placed. The cylinder *C* is connected to a force pump and filled with water. The force pump is worked by an engine, and the load *W* in the iron vessel is raised. This load exerts a great pressure on the water, and this pressure can be transmitted to any other place by strong iron pipes. If the load is one ton-weight, and the area of the ram is 4 square inches, this will give a pressure of 2240 lbwt. per 4 sq. inches, *i.e.* 560 lbwt. per sq. inch.

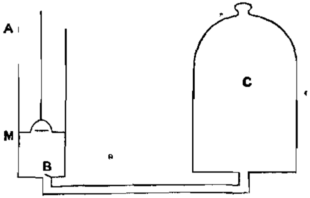
The **hydraulic lift** is built on the same principle. The ram *PP* moves in a cylinder *C*, which is sunk into the ground. The cage is placed on the top of the ram. The pressure is gained by supplying water to the cylinder from a tank placed at a considerable height, or from the main (if it is at high pressure). To lower the cage the water is disconnected from the supply and allowed to flow away.

(iii) The force pump is also used to work the hydraulic presses. The application to Bramah's press is described on page 194.

2. An **Air Pump** is an instrument for exhausting air from any vessel. For many experiments the vessel is a glass bell-jar, and is called a *receiver*.

(i) *To explain the action of Hawksbee's Air Pump.*

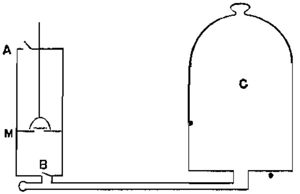
Hawksbee's Air Pump consists of a cylinder *AB*, in which a piston *M* is worked. A tube leads from the bottom at *B*, and is connected with the receiver. In



the piston and at the bottom of the cylinder are valves opening upwards. Suppose the instrument to be full of air, and the piston at *B*. When the piston is raised the air in *BM* becomes rarer (and therefore at less pressure) than the air in the receiver, the valve at *B* is opened, and part of the air passes into *BM*. As the piston descends the air in *BM* becomes denser than the air in the receiver and closes the valve at *B*, and, becoming denser than the atmospheric air, opens the valve at *M* and rushes out. At each stroke a quantity of air is driven out of the receiver. The action will continue until the pressure of the air in the cylinder, when the piston is at the bottom, is not great enough to overcome the pressure of the atmosphere and the weight of the valve.

(ii) *To explain the action of Smeaton's Air Pump.*

Smeaton's Air Pump consists of a cylinder *AB*, in which a piston *M* is worked by a rod passing through an air-tight collar at the top. A tube leads from the bottom at *B*, and is connected to the vessel to be exhausted (called the receiver). At the top of the cylinder, in the piston, and at the bottom of the



cylinder are valves opening upwards. Suppose the instrument to he full of air, and the piston at *B*. When the piston is raised the air in *BM* becomes rarer (and therefore at less pressure) than the air in the receiver, the valve at *B* is opened, and part of the air passes into *BM*. As the piston ascends the air in *MA* becomes denser than the air outside, the valve at *A* is opened, and air rushes out through *A*. As the piston descends the air in *BM* becomes denser than either the air in the receiver or tbat in *MA*, closes the valve at *B*, opens the valve at *M*, and rushes into *MA*, and in the upward stroke is driven out through *A*. At each stroke a quantity of air is driven out of the receiver. This action will go on until the pressure of the air in *MA* is not great enough to lift the valve at *A*.

NOTE.—*To find the density and pressure of the air in the receiver after n strokes of the piston.*

Let the volume of the receiver be V.

Let the volume of the cylinder be V'.

Let ρ be the density of the air in the receiver before commencement of first stroke, the piston being down.

When the piston is drawn up the mass of air which occupied the volume V will expand and fill a volume $V+V'$. Let the new density be ρ'.

Now, the mass of air being the same,

$$(V+V')\rho' = V\rho.$$

$$\therefore \rho' = \frac{V}{V+V'}\rho.$$

When the piston descends the valve at B closes, and the air in the receiver remains at density ρ' during the downward part of the stroke.

$$\text{Hence the density at end of first stroke} = \frac{V}{V+V'}\rho;$$

$$\text{Similar density at end of second stroke} = \frac{V}{V+V'}\rho';$$

$$= \left(\frac{V}{V+V'}\right)^2 \rho.$$

And hence it is easily seen that the density at end of nth stroke is

$$\left(\frac{V}{V+V'}\right)^n \rho.$$

It is thus seen that the densities of the air in the receiver at end of each successive stroke form a geometrical progression.

By Boyle's Law the pressure of a gas is directly proportional to its density.

Hence, if p is the pressure of the air at the beginning, the pressure after n strokes is

$$\left(\frac{V}{V+V'}\right)^n p.$$

Ex.—If the volume of the receiver is twice the volume of the cylinder, what will be the density of the air in the receiver at the end of the third stroke? And about how many strokes will be required to reduce the density to a twentieth of its original density?

Here $V = 2V'$.

$$\therefore \frac{V}{V+V'} = \frac{2V'}{2V'+V'} = \tfrac{2}{3}.$$

Therefore, if ρ is the density of the air in the receiver before the beginning of the first stroke,

at the end of third stroke the density $= (\frac{2}{3})^3\rho = \frac{8}{27}\rho$;

at the end of seventh stroke the density is $(\frac{2}{3})^7\rho = \frac{128}{2187}\rho$;

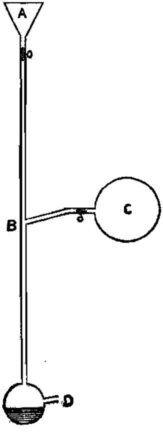
at the end of eighth stroke the density is $(\frac{2}{3})^8\rho = \frac{256}{6561}\rho$.

Hence the density at the end of seventh stroke is greater than $\frac{1}{20}$ of original density, and at the end of eighth stroke is less than $\frac{1}{20}$.

The density is therefore $\frac{1}{20}$ of original density between the seventh and eighth stroke.

(iii) *To explain the action of Sprengel's Air Pump.*

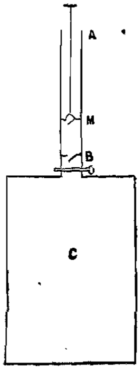
Sprengel's Air Pump consists of a long narrow tube, into which leads another tube which is connected



with the receiver. At the top of the tube a funnel is placed, and the bottom of the tube opens into a vessel, which is provided with a spout at D. A continuous stream of mercury is poured down the tube, and as each drop of mercury passes the entrance of the other tube at B it carries away with it a small portion of the air from the receiver. As the mercury passes out at D it is replaced into the funnel. As there are no valves to open, a greater exhaustion can be obtained by this means than by the preceding air pumps.

(iv) *To explain the action of an Air Condenser.*

A Condenser is an instrument used to condense air in a receiver. Its action is just the reverse of that of an air pump. A cylinder AB is connected with a receiver C. A piston M, fitted with a valve opening downwards, works air-tight in the cylinder. At the bottom of the cylinder is a valve B opening downwards.



Suppose the instrument to be full of air. As the piston descends the air in BM becomes denser (and therefore at greater pressure) than the air outside, closes the valve at M, opens the valve at B, and is forced into the receiver. As the piston ascends the air in

BM becomes rarer than the air in *C*, the valve at *B* is closed, and the air in *BM* becoming rarer than the air outside, the valve at *M* opens, and more air passes into the cylinder. Every downward stroke drives a quantity of air into the receiver, and at every upward stroke more air passes into the cylinder. The action goes on as long as there is sufficient force to drive the piston.

NOTE 1.—*To find the density of the air in the receiver after .. strokes of the piston.*

Let the volume of the receiver be V.

Let the volume of the cylinder be V'

Let the density of the atmosphere be ρ.

At each downward stroke a quantity of air of volume V' and density ρ is driven into the receiver.

Hence the mass of air which enters the receiver at each stroke is $V'\rho$.

The mass of air forced into the receiver by n strokes is $nV'\rho$.

The mass of air in the receiver at first is $V\rho$.

$\therefore$ mass of air in receiver at end of nth stroke is $V\rho + nV'\rho$.

If ρ' be the density of this air, then

$$V\rho' = V\rho + nV'\rho;$$
$$\rho' = \frac{V + nV'}{V}\rho.$$

NOTE 2.—To measure the pressure, and hence the density, of the air in a receiver a gauge is used.

This gauge can have any of the forms described in the previous chapter. An Air Pump for experimental purposes is generally fitted with a closed tube vacuum gauge, called a *manometer*. As it is only necessary to measure low pressures, the arms of the gauges are short. The manometer is either fitted into a strong glass tube which communicates by means of a pipe with the receiver, or the open branch is screwed air-tight to a pipe leading into the receiver.

EXERCISES.

1. The cylinder of an air pump contains 10 cubic inches, and it is used to exhaust a vessel which contains 100 cubic inches of air at the pressure of the standard atmosphere. Find the pressure of the air in the vessel after the fourth

stroke. Find also the mass of the air in the vessel at the end of the fourth stroke.

2. The cross section of the piston of an air pump is 2 square inches, and the stroke is 4 inches. The receiver contains 56 cubic inches of air at a density of 1·29 gm. per litre. Find the density at the end of the third stroke.

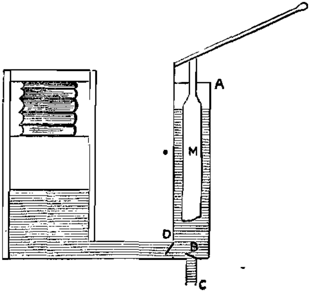
3. The volume of a receiver is twice the volume of the cylinder of an air pump. After how many strokes will the density in the receiver be reduced to one-fifth of its original density?

4. If the valve at the bottom of the cylinder of the air pump in Question 3 requires a pressure of at least one-thirtieth of the pressure of the atmosphere to lift it, after how many strokes will the best possible vacuum be obtained?

5. The volume of the cylinder of a condenser is one-tenth the volume of the receiver. Find the mass of air in the receiver at the end of the fiftieth stroke. The mass at first is 1 gm., and the pressure 760 mm. of mercury. Find also the pressure at the end of the fiftieth stroke.

3. The **Hydraulic or Bramah's Press** consists of a force pump, the tube of which leads into a large cylinder, in which moves a smaller cylinder or ram in a water-tight collar. The piston of the pump is worked by a lever, and by the principle of transmissibility of fluid pressure the pressure communicated to the water by the piston is transmitted without change to the cylinder. If the area of the cylinder is twenty times that of the piston, then, since the force on every square inch of the cylinder is the same as the force on every

square inch of the piston, the force on the cylinder must be twenty times the force on the piston. In this way a small force applied to the piston will produce a large force on the cylinder.



NOTE 1.—*To find the mechanical advantage of the Hydraulic Press.*

Let the area of the piston be A square inches, and the force exerted on it P lbwt.

Let the area of the cylinder be B square inches, and the load on it W lbwt.

The pressure exerted by the piston on the water is $\frac{P}{A}$ lbwt. per sq. inch, and this is communicated to the cylinder.

$\therefore$ the force exerted by the water on the cylinder is $\frac{P}{A} \times B$ lbwt.

And if this just supports the load W lbwt., $W = \frac{P}{A} \times B$.

$\therefore$ the mechanical advantage is $\frac{W}{P} = \frac{B}{A}$.

The mechanical advantage can be found directly from the principle of **Virtual Work**.

Suppose the piston is driven down 1 foot, then, if the area of the piston is A square feet, a volume of A cubic feet of water is displaced. Since the water is not appreciably compressed, the cylinder must be lifted; and if its area is B square feet, it will be lifted $\frac{A}{B}$ feet.

If the force applied to the piston is P lbwt., the work done by it is P ft.-lbwt.

If the load lifted is W lb., the work done in lifting it $\frac{A}{B}$ feet is $W \times \frac{A}{B}$ ft.-lbwt.

And, neglecting the work required to overcome the friction, the work due to the force acting on the piston is equal to the work done in lifting the load.

$$\therefore P = W \times \frac{A}{B}.$$

And the mechanical advantage $= \frac{B}{A}$.

Ex.—The arms of the lever are as 4 : 1, the area of the piston is 1 square inch, and the area of the cylinder is 3 square feet. Find the force which can be exerted by the application of a force of 60 lbwt. at the end of the lever.

A force of 60 lbwt. at the end of the lever produces a force of 60×4 (*i.e.* 240) lbwt. on the piston.

The area of the cylinder is 3 square feet, *i.e.* 3×144 square inches, *i.e.* 432 square inches, so that it is 432 times the area of the piston.

The force on the cylinder is therefore 432 times the force on the piston, *i.e.* it is 432×240 lbwt. $= 46\frac{2}{7}$ ton-wt.

NOTE 2.—The pressure of the water in a Bramah press is often very great, so that the collar through which the cylinder or ram works must be made capable of resisting great pressure, and at the same time offering as little frictional resistance as the nature of the case will allow. This is generally done by means of a cup-leather packing, a sectional drawing of which is seen surrounding the ram

of the hydraulic accumulator on page 187. This consists of a ring of leather, the section of which is an inverted U, which is kept in place by a ring of brass which is screwed down upon it. When the cylinder is driven upwards, the water is forced into the U cup, and its sides are pressed against the ram and the cylinder, and thus prevents the water leaking out. The greater the pressure the more firmly is the ring pressed against the sides.

EXERCISES.

1. The diameters of the piston and ram of a press are respectively 2 inches and 20 inches, and the lengths of the arms of the lever are 2 inches and 18 inches. Find the load which can be supported by a force of 28 lbwt.

2. The ram of an hydraulic accumulator is 1·5 inches diameter, and the load is half a ton.* Find the force which will be exerted on a piston of a lift the diameter of which is 4 inches.

3. If in the pump on page 186, $AC = 2$ feet, $AB = 4$ inches, what must be the diameter of the plunger in order that a force of 56 lbwt. may produce a pressure of 200 lbwt. per square inch throughout the water? If the pipe E is connected to a press, what must be the diameter of the ram to produce a force of 100 ton-wt.?

4. A cage, the weight of which is balanced, is constructed to carry ten persons of average weight 140 lbwt. If the water is supplied from a tank 80 feet high, what must be the diameter of the ram?

EXAMPLES ON CHAP. VI.

1. The diameter of the piston of a common pump is inches, and it lifts water a height of 20 feet. Find t force exerted on the piston.

2. A force pump is used to draw water from a depth of 15 feet, and drive it to a height of 60 feet. If the diameter of the plunger is 10 inches, find the force along the piston rod in the back and forward stroke.

3. A force pump is used to supply water to a steam boiler. If the diameter of the plunger is 4 inches, find the force on the piston rod in the forward stroke when the pressure in the boiler is 30 lbwt. per sq. inch.

4. A syringe is formed of a piece of glass tubing of uniform bore. When the piston is at its lowest position the length of tube below it is 4 inches, how high will the water rise when the piston is drawn up 3 inches, the height of the water barometer being 33 feet, and the mouth of the syringe being kept just below the surface of the water?

5. If the section of the pipe is 12 square inches, that of the barrel 192 square inches, the height of the lower valve above the surface of the water 20 feet, and the length of the stroke 1 foot, find the height to which the water rises each stroke until it enters the barrel, and the force on the piston rod at the beginning and end of each stroke.

6. The diameter of the plunger of a force pump is 8 inches, and the stroke is 6 inches. If the pump is worked at the rate of 120 strokes a minute, find the volume and mass of water delivered in one hour.

7. If the length of the lower pipe of a water pump above the surface of the water be 16 feet, and the area of the upper pipe 16 times that of the lower, find the length of stroke that water may just rise into the upper pipe at the end of the first stroke, the water barometer standing at 32 feet.

8. The capacity of the receiver of a Smeaton's air pump is [illegible] that of the barrel. Show that the upper valves will open during the third upward stroke when the piston has made [illegible] of its ascent.

9. If the piston of an air pump have a range of 6 inches, and at its highest and lowest positions be one-fourth of an

nch from the top and bottom of the barrel respectively, how that the density of the air in the receiver cannot be educed below $\frac{1}{25}$ of the atmospheric density.

10. If the capacity of each barrel of a double-barrelled ir pump be one-fourth that of the receiver, and the mer-urial gauge at the commencement of the operation stands t 30 inches, what will be the least number of strokes vhich will reduce the height of the gauge to 5 inches?

11. The receiver of an air pump contains 379 cubic nches, the cylinder 58 cubic inches. After how many trokes will the density be reduced to one-tenth?

12. The contents of a receiver is six times that of the arrel. Find the pressure of air in the receiver at the end of he *eighth stroke of the piston. An atmosphere is taken* s 15 lbwt. per square inch.

13. The area of the piston of a force pump is 1 square oot. What force must be exerted on the piston to raise vater to 40 feet?

14. If a vessel of water containing a floating body be laced under the receiver of an air pump, and the air radually exhausted, describe in general terms the effect on he position of the body in the water.

15. The volume of the receiver of an air pump being ight times that of the barrel, after how many strokes will he density of the air in the receiver be half that of the xternal air?

16. A tank on the sea-shore is filled by the tide with ea water whose specific gravity is 1·025. It is desired to mpty it at low tide by means of a common pump whose ower valve is on the same level as the top of the tank. ind the greatest depth which the tank can have so that his may be possible when the water barometer stands at 4 ft. 2 in.

17. What alteration would be required in a condenser the piston be replaced by a solid piston without a valve?

18. The volume of the receiver of a condenser being eight times that of the barrel, after how many strokes will the density of the air in the receiver be twice that of the external air?

19. In one air pump the volume of the barrel is one-tenth that of the receiver, and in another it is one-fifth of it. Show that the densities of the air in the two receivers after three ascents of the piston are as $12^3 : 11^3$.

20. The section of the pipe of a common pump is 36 square inches, the section of the barrel 1 square foot, the length of the pipe is 24 feet, and the water rises half way up the pipe at the end of the first stroke. Find the length of the stroke, the water barometer standing at 33 feet. How far will the water have risen when the piston has been raised 3 feet? How many strokes will be required to raise the water into the barrel? Show that it will rise to within ·58 foot of the barrel at the end of the second stroke.

21. A glass tube of uniform bore is fitted with an air-tight piston, and its open end placed in water. If the length of the tube below the piston occupied by air is 4 inches, through what distance will the piston have to be drawn so that the water may rise 10 inches up the tube? Height of the water barometer 33 feet.

22. If the tube in Question 21 were placed in oil of specific gravity ·75, through what height would the piston have to be drawn so that the oil would rise to a height of 10 inches?

23. The stroke of a common pump is 8 inches, the diameter of the barrel is 4 inches, and the diameter of the pipe is 1 inch. If the lower valve is 15 feet above the level of the water, to what height will the water rise at the end of the first stroke?

24. To lift one of the tubes of the Conway bridge two presses were used, the rams of which had a diameter of 18 inches and the pistons $1\frac{1}{16}$ inches. What was the force applied, the weight of each tube being 630 tons-wt.?

25. One of the presses used in lifting the tubes of the Britannia Bridge was constructed to lift 1144 tons, the diameter of the ram was 20 inches, and that of the piston $2\frac{1}{2}$ inches. What force would have to be applied?

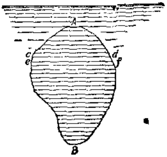
26. A lift constructed to carry 15000 lb. is supplied with water from a height of 164 feet. Find the necessary diameter of the ram.

27. A lift constructed to carry 50 persons of average weight 10 stones is worked by a ram 30 inches diameter. If the pressure is derived from an accumulator, the ram of which is 1 inch diameter, what must be the load on the accumulator?

CHAPTER VII.

Force on any Surface Immersed in a Liquid.

Prop. 1.—*To find the magnitude of the force which a liquid exerts on a plane area.*



Let AB be the plane area, immersed in a liquid with its plane vertical.

Divide the area into a large number of very narrow strips by lines parallel to the surface of the liquid.

If the strips are extremely narrow, the pressure will be the same throughout each strip.

Let $z_1, z_2, z_3, \ldots$ be the depths of these strips.

Let w be the weight of the unit volume of the liquid.

Then the pressure at any point of the strips will be respectively

$$wz_1,\ wz_2,\ wz_3,\ \ldots.$$

Let $a_1, a_2, a_3, \ldots$ be the areas of the strips.

The forces on the strips are $wz_1a_1,\ wz_2a_2,\ wz_3a_3,\ \ldots.$

Hence, since these forces are all perpendicular to the plane, their resultant is found by adding them together.

Hence the resultant force of the liquid on the plane

$$= wz_1a_1 + wz_2a_2 + wz_3a_3 + \ldots;$$

$$= w\{z_1a_1 + z_2a_2 + z_3a_3 + \ldots\} \ldots\ldots\ldots\ldots(\text{i})$$

If z = the depth of the centre of gravity of the area, it is proved in Statics (by taking moments about the line in which the plane produced will intersect the surface) that

$$Az = z_1a_1 + z_2a_2 + z_3a_3 + \ldots\ldots\ldots\ldots\ldots\ldots(\text{ii})$$

where A is the area of the plane.

$\therefore$ from (i) the resultant force $= wAz.$

Therefore the resultant force of a liquid on a plane is equal to the weight of water contained in a cylinder, the base of which is equal to the area of the plane, and the length of which is equal to the depth of the centre of gravity of the plane.

Ex 1.—Find the force on one side of a rectangle, the length of which is 100 feet and height 10 feet. The rectangle is placed in a vertical position with its upper edge in the surface of water.

The area of the plane is 100×10, *i.e.* 1000 sq. feet.
The depth of its centre of gravity is 5 feet.
∴ the force on one side
$=$ weight of 1000×5, *i.e.* 5000 cubic feet of water;
$= 5000 \times 1000$ ozwt. $= 312500$ lbwt.

Ex. 2.—The base of a triangle is 6 feet and height 4 feet. It is placed with its plane vertical, and its base horizontal at a depth of 1 foot below the surface of water. Find the force on one side of it.

The area of the triangle $= \frac{6 \times 4}{2}$ sq. feet $= 12$ sq. feet.
The depth of its centre of gravity $= 1 + \frac{4}{3} = \frac{7}{3}$ feet.
∴ the force of the water on one side
$=$ weight of $12 \times \frac{7}{3}$ cubic feet of water;
$= 12 \times \frac{7}{3} \times 1000$ ozwt. $= 1750$ lbwt.

DEF. **The Centre of Pressure of a plane immersed in a liquid is the point in the plane through which the direction of the resultant force of liquid on the plane passes.**

PROP. II.—*To find the depth of the centre of pressure of any plane immersed in a liquid.*

Let AB in PROP. I. represent the plane, and suppose it divided as in that proposition.

The forces on the strips are respectively

$$wz_1a_1,\ wz_2a_2,\ wz_3a_3, \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

and the depths of the points in the planes through which these forces act are respectively

$$z_1,\ z_2,\ z_3 \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

The resultant force is wAz.

Take moments about the line in which the plane produced intersects the surface.

The sum of the moments of the forces on the strips

$$= wa_1z_1^2 + wa_2z_2^2 + wa_3z_3^2 + \ldots\ldots\ldots\ldots \text{(iii)}$$

If h is the depth of the centre of pressure, *i.e.* the depth of the point through which the resultant force wAz acts, the moment of the resultant of the forces (i) $= wAzh$.

$$\therefore wAzh = wa_1z_1^2 + wa_2z_2^2 + wa_3z_3^2 + \ldots;$$

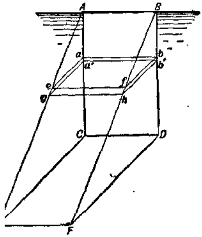
$$\therefore h = \frac{a_1z_1^2 + a_2z_2^2 + a_3z_3^2 + \ldots}{Az}.$$

The application of this form generally requires the Integral Calculus.

A special method for finding the C.P. of the rectangle and triangle and figures which can be divided into rectangles and triangles will be given in the following propositions.

PROP. III.—*To find the depth of the* C.P. *of a rectangle with one edge in the surface.*

Let $ABCD$ be the rectangle which is placed vertically in a liquid with the side AB in the surface.



The pressure at any point of the rectangle is proportional to its depth below the surface. Draw DF perpendicular to the plane and equal to BD. Draw CE perpendicular to the plane and equal to FD. Join AE, BF, EF, thus forming the wedge $ABCDEF$. If the plane $ABCD$ be divided into a very large number of thin

strips, as at $aba'b'$, then the force on this strip is equal to $w \cdot Bb \times$ area of $aa'bb'$. But $Bb = bf$, and $bf \times$ area of $aa'bb'$ is equal to the volume of the slice of the wedge $aa'f$. Hence the force on ab' is equal to the weight of the slice af. Hence, if we imagine the wedge turned so that $ABCD$ is horizontal and forms the base of the wedge, the force on it due to the weight of the wedge is the same as the force of the fluid on it in the vertical position.

But the weight of the wedge will act through the C.G. of the wedge, and as the wedge can be divided into a series of parallel triangular sheets parallel to ACE, the distance of the C.G. of the wedge from the plane $CDEF$ must be one-third of BD. Hence the weight of the wedge will act through a point in the plane at a distance from AB equal to two-thirds of BD.

Therefore the force which the liquid exerts on the plane must pass through a point in the plane at a depth equal to two-thirds of BD.

Hence the depth of the centre of pressure of a rectangle, with one edge in the surface (at which there is no pressure) is two-thirds the depth of the lower horizontal side.

NOTE.—This method can be applied to a plane of any form; but the wedge-shaped figure will not generally be one the centre of gravity of which can be found by elementary methods.

The force of the liquid on the rectangle is equal to the weight of the liquid wedge $ABCDEF$.

Let the horizontal side $= a$, vertical side $= b$.

Since the wedge is a prism on the base BFD and of the height AB its volume is $\frac{b^2}{2} \times a$.

$$\therefore \text{ force on the plane} = \text{weight of } \frac{b^2a}{2} \text{ of liquid;}$$

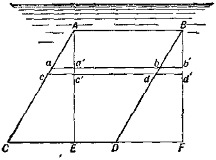
$$= \text{weight of } ba \times \frac{b}{2} \text{ of liquid.}$$

But $ba =$ area of the rectangle, and $\frac{b}{2} =$ depth of C.G. of rectangle.

$\therefore$ as in PROP. I. the force on the rectangle $=$ weight of the liquid which would be contained in a cylinder having the plane as base and the depth of its centre of gravity as height.

COR. I.—*The depth of the* C. P. *of a parallelogram is the same as that of the rectangle on the same horizontal base and between the same parallels.*

Let $ABCD$ be the parallelogram. Construct the rectangle $ABEF$ on same base AB and between the same parallels AB, CD. Then, if the parallelogram and rectangle are divided into



parallel strips as $abcd$, $a'b'c'd'$, the area of each such strip $abcd$ of the parallelogram is equal to the area of the corresponding strip $a'b'c'd'$ of the rectangle, and their average depths are equal; and therefore the force of the liquid on $abcd$ is equal to the force of the liquid on $a'b'c'd'$, and acts at the same depth. Hence, by taking moments about AB, it is clear that the depth of the point through which passes the resultant of the forces acting on the strips of the parallelogram must be the same as the depth of the point through which the resultant of the forces on the strips of the rectangle passes.

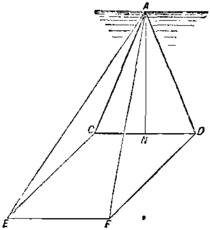
Therefore the depth of the C. P. of the parallelogram is equal to the depth of the C. P. of the rectangle.

COR. II.—*If a plane immersed in a liquid be rotated about the line in which it cuts the surface, the centre of pressure is at the same point in the plane, at whatever angle it makes with the vertical.*

The proof in PROP. III. holds for the case where the planes make an angle with the vertical. The resultant force of the liquid on the plane is equal to the weight of the wedge of liquid, and its direction is perpendicular to the plane and passes through the centre of gravity of the wedge, and hence passes through a point in the plane which is independent of the angle it makes with the vertical.

PROP. IV.—*To find the depth of the centre of pressure of a triangle having an angular point in the surface and the opposite side horizontal.*

Let ACD be the triangle with the point A in the surface and the side CD horizontal.

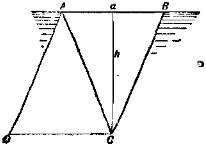


Draw DF, CE perpendicular to the plane and equal to the depths of D and C below the surface. As in Prop. III. form the wedge $ACDEF$. Therefore the force of the liquid in ACD is equal to the weight of this wedge, and is perpendicular to the plane, and passes through the centre of gravity of the wedge.

Now, the wedge is a pyramid, and the centre of gravity is at depth below A equal to $\frac{3}{4}$ AN, where AN is the perpendicular from A on to the base of the pyramid.

Hence the depth of the centre of pressure of such a triangle is at a depth equal to three-fourths the depth of the horizontal side.

Prop. V.—*To find the depth of the centre of pressure of a triangle with one side in the surface.*



Let ABC be the triangle with the side AB in the surface. Complete the parallelogram $ABCD$.

Now, the force on ABC is the resultant of the force on $ABCD$ and of the force on ADC acting in the opposite direction.

Let $AB=a$, the perpendicular $Ca=h$.

$$\text{Force on } ABCD = wah \times \frac{h}{2} = \frac{wah^2}{2} \ldots(\text{PROP. III.})$$

and it acts at a depth of $\frac{2}{3}h$.........(PROP. III.)

$$\text{Force on } ADC = \frac{wah}{2} \times \frac{2}{3}h = \frac{wah^2}{3};$$

and it acts at a depth of $\frac{3}{4}h$ ......(PROP. IV.)

$$\text{Force on } ABC = \frac{wah}{2} \times \frac{h}{3} = \frac{wah^2}{6};$$

and if z is the depth of the C.P. of ABC, then (by taking moments about AB)

$$\frac{wah^2}{6} \times z = \frac{wah^2}{2} \times \frac{2h}{3} - \frac{wah^2}{3} \times \frac{3h}{4}.$$

$$\therefore z = 2h - \tfrac{3}{2}h;$$

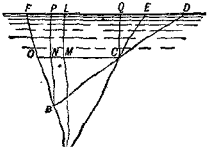
$$= \frac{h}{2}.$$

Hence the depth of the centre of pressure of a triangle with one side in the surface is half the depth of the opposite angular point.

PROP. VI.—*To find the depth of the centre of pressure of any triangle.*

Let ABC be the triangle.

Produce its sides to meet the surface in E, D, F.



Draw the perpendiculars $AL\,(=x)$, $BP\,(=y)$, $CQ\,(=z)$.
The $\triangle ABC = \triangle AFE + \triangle ECD - \triangle BFD.$

$\therefore$ force on $\triangle ABC$ is the resultant of

(1) Forces on AFE and ECD acting in one direction; and

(2) Force on BFD acting in the opposite direction.

$$\text{Force on } AFE = w \times \frac{FE \times x}{2} \times \frac{x}{3} = \frac{w \,.\, FE \,.\, x^2}{6}$$

and it acts at a depth of $\frac{x}{2}$.

$$\text{Force on } ECD = \frac{w \,.\, ED \,.\, z^2}{6},$$

acting at a depth of $\frac{z}{2}$.

$$\text{Force on } BFD = \frac{w \,.\, FD \,.\, y^2}{6},$$

acting at a depth of $\frac{y}{2}$.

Resultant of these, *i.e.* force on ABC,

$$= \frac{w \,.\, FE \,.\, x^2}{6} + \frac{w \,.\, ED \,.\, z^2}{6} - \frac{w \,.\, FD \,.\, y^2}{6},$$

d if z' be the depth of the centre of pressure of the triangle ABC, taking moment about FD we get

$$z'\left\{\frac{w \,.\, FE \,.\, x^2}{6} + \frac{w \,.\, ED \,.\, z^2}{6} - \frac{w \,.\, FD \,.\, y^2}{6}\right\}$$
$$= \frac{w \,.\, FE \,.\, x^2}{6} \times \frac{x}{2} + \frac{w \,.\, ED \,.\, z^2}{6} \times \frac{z}{2} - \frac{w \,.\, FD \,.\, y^2}{6} \times \frac{y}{2}.$$
$$\therefore z' = \tfrac{1}{2}\,\frac{FE \,.\, x^3 + ED \,.\, z^3 - FD \,.\, y^3}{FE \,.\, x^2 + ED \,.\, z^2 - FD \,.\, y^2} \quad \text{..........(i)}$$

Draw $CMNO$ parallel to the surface as in the figure.

By similar triangles

$$\frac{FE}{CO} = \frac{AL}{AM} = \frac{x}{x - z};$$
$$\frac{FD}{CO} = \frac{BP}{BN} = \frac{y}{y - z}.$$
$$\therefore \frac{FE}{FD} = \frac{x\,(y - z)}{y\,(x - z)}.$$

$$\therefore \frac{FE}{x(y-z)} = \frac{FD}{y(x-z)}.$$

And in a similar manner it can be shown that

$$\frac{FE}{x(y-z)} = \frac{ED}{z(x-y)}.$$

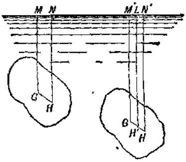
$\therefore$ substituting in (i)

$$z' = \tfrac{1}{2} \cdot \frac{x^4(y-z) + y^4(z-x) + z^4(x-y)}{x^3(y-z) + y^3(z-x) + z^3(x-y)};$$

$$= \tfrac{1}{2} \cdot \frac{x^2 + y^2 + z^2 + xy + yz + zx}{x + y + z}.$$

PROP. VII.—*Given the depth of the centre of pressure of a plane below the surface of a liquid, to find the centre of pressure when every point of the plane is depressed a further depth* h *below the surface.*

Let G be the C.G. of the plane; H its C.P. in the first position.



Suppose the plane depressed a depth h so as to occupy the position of the second figure.

The force on the area in the lower position is equal to the force in the first position + the force due to the increase of depth.

Since the increase of pressure is uniformly wh throughout the area, the additional force on the area is equal to whA where A is the area of the plane; and since the pressure is uniform this additional force will act through the C.G. of the area, viz. at G.

Hence the force on the area is due to

(1) whA at G;

(2) $w \,.\, GM \,.\, A$ at H.

If this force passes through H', then (taking moments about $M'N'$) (if the perpendicular $H'L = z$)

$$(whA + w \,.\, GM \,.\, A) z = whA \times GM' + w \,.\, GM \,.\, A \times HN'.$$

$$\therefore z = \frac{h \,.\, GM' + GM \,.\, HN'}{h + GM}.$$

Ex.—Find the depth of the C.P. of the rectangle in PROP. III. if there is a pressure Π at the surface.

Let h be the head of water equivalent to the pressure Π.

$$\therefore \Pi = wh.$$

$\therefore$ the increase of pressure is equivalent to an increase of depth h.

Then from the Proposition, since in this case

$$GM = \frac{b}{2}; \quad \therefore GM' = \frac{b}{2} + h.$$

$$HN = \frac{2b}{3}; \quad \therefore HN' = \frac{2b}{3} + h.$$

$$z = \frac{h\left(\frac{b}{2} + h\right) + \frac{b}{2}\left(\frac{2b}{3} + h\right)}{h + \frac{b}{2}};$$

$$= h + \frac{\frac{b}{2}\left(\frac{2b}{3} + h\right)}{h + \frac{b}{2}}.$$

$\therefore$ the depth of the centre of pressure is

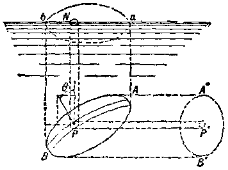
$$z - h = \frac{3hb + 2b^2}{3(2h + b)}.$$

PROP. VIII.—*To find the vertical component and horizontal component in a given direction of the force which a liquid exerts on any surface.*

(i) *To find the vertical component.*

Let AB be the surface. Divide the surface into a very large number of small areas, so that each area is practically a small plane. Let a be the area of one of these at P. Then the force which the

liquid exerts upon this area is perpendicular to the area, and is equal to $w \cdot a \cdot PN$, where PN is the depth of P.



If the angle the direction of this force makes with the vertical is θ, the vertical component of the force $= w \cdot a\, PN \cdot \cos\theta$.

But $a \cos\theta =$ the area of the cross section of the cylinder PN drawn with vertical sides through the boundary of the small plane.

$\therefore$ $wa \cos\theta\, PN =$ weight of the liquid contained in the cylinder PN.

Hence the vertical component of the force on the area AB is equal to the weight of the liquid contained in the cylinder $ABab$, *i.e.* equal to the weight of the liquid vertically above the surface AB. This force acts downwards.

In the same way the upward vertical force on a surface can be found.

(ii) *To find the horizontal component in a given direction.*

Let PP' be the given direction.

Let $A'B'$ be the projection of the surface AB on the plan pendicular to PP'.

Then, if the angle the normal to the plane at P makes with PP' is ϕ, the horizontal component of the force on the small area at P is $w\,PNa \cos\phi$.

But $a \cos\phi$ is the projection of the area at P on the plane $A'B'$.

The pressure at $P' = w \cdot P'N' = w \cdot PN$.

$\therefore$ the force on the projection of $a = w \cdot PNa \cos\phi$,

∴ the horizontal component of the force on the small area at $P =$ the force on its projection at P'.

Hence, by adding together the horizontal components of the forces on each small area of the surface AB, the horizontal component of the force on AB in the direction of $PP' =$ the force on the plane $A'B'$.

This force can be found from PROP. I., and the depth of its line of action can be found from PROP. II., as its depth is the depth of the C.P. of the vertical plane $A'B'$.

The general proposition of Chapter IV. may be proved by a direct application of this proposition.

EXAMPLES ON CHAP. VII.

1. Find the force of water on the following areas immersed with their planes vertical:

(*a*) A square of 2 feet side, with one side in the surface.

(*b*) A square of 2 inches side, with an angular point in the surface and a diagonal horizontal.

(*c*) An oblong 8 feet long, 4 feet deep, with one of its longer sides 6 inches above the surface.

(*d*) A circle 4 inches diameter, placed with its centre at a depth of 8 feet.

2. A triangle, whose base is 85 cm. and height 40 cm., is placed vertically with its base horizontal and vertex 20 cm. below the surface of mercury (sp. gr. 13·6). Find the force on the triangle in the two possible positions.

3. A cubical box, the edge of which is 10 inches, is filled with water. Find the force on each of the sides of the cube.

4. A rectangle 10 feet long and 4 feet broad is placed with its plane vertical, and one of its long edges in the surface of water. If it is free to turn about this edge, find

the force which must be applied horizontally at the lower edge in order to keep the rectangle in its vertical position.

5. The difference of level of the water on two sides of lock gates is 10 feet, and each gate is 5 feet wide. What is the resultant force on each? and if (in plan) they are inclined at an angle of 120° to each other, what is the force between their edges?

6. A reservoir has its sides sloping at an angle of 30° to the horizontal. The depth of the water is 50 feet. Find the force on one side of the reservoir, which is 300 yards long.

7. A regular hexagon of 2-feet side is placed vertically with one side in the surface of water. Find the force of the water on it and the depth of its line of action.

8. A cube of 8 inches side is sunk in water until its top, which is horizontal, is 30 inches below the surface. Find the force on each side (*a*) neglecting the atmospheric pressure (*b*) when the atmospheric pressure is 33 feet of water.

APPENDIX I.

1. *To prove that the pressure at any point of a fluid at rest is the same in all directions about the point.*

Consider the equilibrium of a very small isosceles triangular prism of the fluid. This prism will be kept at rest by the forces on its plane surfaces and by its weight. Since the force on each face of the prism is proportional to the *area* of the face, and the weight of the prism is proportional to the *volume* of the prism, when the size of the prism is very small the weight of the prism will be indefinitely small compared with the force on each face, and therefore the prism may be considered at rest under the action of the forces on the faces only. Since there is no motion in the direction of the edge, the forces on the three faces of the prism are in equilibrium; and the direction of the forces on the equal faces being equally inclined to the force on the third face, by the "triangle of forces" these forces are equal; and therefore, if the prism be diminished indefinitely, the pressures at a point, the directions of which are perpendicular to the faces, are equal, *i.e.* the pressures at a point in two given directions are equal. Now, the equal faces may be inclined at any angle, therefore the pressure at any point of a fluid at rest is the same in all directions about the point.

NOTE.—That the weight of the prism compared with the force on each face may be neglected in the proof is made clear by the following considerations: Let a be the length of each sides of the isosceles triangle, θ the angle between these sides, and l the length of the edge of the prism. The area of each equal face is al, and if p is the pressure at any point of one of the faces, the force on the face is pal. The volume of the prism is $\frac{1}{2}a^2l \sin\theta$, and if w is the weight of unit volume of the fluid, the weight of the prism is $\frac{1}{2}wa^2l \sin\theta$. The ratio of the weight of the prism to the force on a face is $\frac{1}{2}wa \sin\theta \div p$, which is indefinitely small, when the prism,

and therefore a, is diminished indefinitely, *i.e.* the weight of the prism is ultimately indefinitely small compared with the force on each face.

2. The variation in the force of gravity has been neglected in the text. The force on a given mass due to the attraction of the earth (*i.e.* the weight of the given mass) varies both in direction and magnitude. The direction is always in the line joining the centre of gravity of the body to the centre of the earth (*i.e.* in the direction of the vertical of the centre of gravity of the body), and the magnitude is inversely as the square of the distance from the centre of the earth.

(i) PROPS. I. III. IV. of Chap. III. are proved under the assumption that the force of gravity is constant in direction. The verticals at two points on the surface of the earth 70 miles apart are inclined at an angle of only 1°, so that the verticals of points within a small area are very approximately parallel. Hence the proofs of Chap. III. are true for surfaces of small area. It follows, therefore, that the surface of a large volume of water, such as a lake, sea, or ocean is spherical, for the surface may be supposed divided into a very large number of very small areas, every one of which is very approximately a plane perpendicular to the vertical of its centre of gravity.

(ii) The force of gravity on a given mass (*i.e.* the weight of the mass) *varies inversely as the square of the distance of the centre of gravity of the mass from the centre of the earth.* This variation in the weight of a substance becomes important in the case of the atmosphere, which *extends to a considerable height above the level of the sea.* In determining the weight of a long vertical column of air, this variation in the weight of unit volume of air must be taken into account.

3. CHARLES' LAW OF A PERFECT GAS. **The volume of a given mass of gas under constant pressure increases uniformly with increase of temperature, and the amount it increases for every Centigrade degree is ·00367 of the volume it occupies at 0° C.**

This law states that the coefficient of cubical expansion of a gas under constant pressure is ·00367 or $\frac{1}{273}$ nearly.

If the volume of a gas is 273 units at 0° C., then for every Centigrade degree decrease of temperature, it decreases $\frac{273}{273}$, *i.e.* 1 unit. Therefore for 273° C. it decreases 273 units. Hence, at a temperature of 273° C. below freezing point of water, *i.e.* at a temperature of — 273° C., the volume of the gas would (*if it continued to obey Charles' Law*) be reduced to zero. This temperature is called the Absolute Zero of the Air Thermometer.

Temperature measured from this Absolute Zero is called Absolute Temperature. Thus a body at a temperature of 15° C. is at an absolute temperature of 273 + 15° C., *i.e.* 288° C. This may be written 288° C. *Ab*. If the volume of a gas is 273 units at 0° C., it is 1 unit at 1° C. *Ab*, 2 units at 2° C. *Ab*, 3 units at 3° C. *Ab*, and so on, so that **the volume of the gas is directly proportional to the Absolute Temperature if the pressure is kept constant.**

This law of direct proportion makes it a simple matter to determine the volume of a gas at any temperature.

PROP. I.—*To find the volume of a gas at* t'° C. *if it occupies a volume of* V cc. *at* t° C.

The absolute temperature of t° C. is $(273 + t)$° C. *Ab*.

The absolute temperature of t'° C. is $(273 + t')$° C. *Ab*.

When the absolute temperature is $273 + t$° C., volume is V cc.

When the absolute temperature is 1° C., volume is $\frac{V}{273 + t}$ cc.

When the absolute temperature is $273 + t'$° C., volume is

$$\frac{273 + t'}{273 + t} \times V \text{cc.}$$

Ex.—A gas occupies 250 cc. at 15° C. Find its volume under the same pressure at 100° C.

Absolute temperature of 15° C. is 273 + 15, *i.e.* 288° C. *Ab*.

Absolute temperature of 100° C. is 273 + 100, *i.e.* 373° C. *Ab*.

When the temperature is 288° C. *Ab*, the volume is 250 cc.

When the temperature is 1° C. *Ab*, the volume is $\frac{250}{288}$ cc.

When the temperature is 373° C. *Ab*, the volume is $\frac{373 \times 250}{288}$ cc.

PROP. II.—*To prove that the pressure of a given mass of gas at constant volume varies directly as the absolute temperature.*

If at a given pressure the absolute temperature of a given mass of gas is doubled, then, according to Charles' Law, the volume is doubled. If the temperature is now maintained the same, *i.e.* double of the original temperature, and the volume is reduced to its original volume, *i.e.* is halved; then, according to Boyle's Law, the pressure is doubled. So that we have doubled the original absolute temperature, and the pressure is doubled, the volume remaining the same as at first. Thus it is seen that, **when the volume is kept constant, the pressure of a given mass of gas varies as the absolute temperature.**

The above may be proved in a more general manner.

Let V_0 be the volume of the gas at 0° C. and pressure p_0.

Let v' be the volume of the gas at t° C. and pressure p_0.

Let v be the volume of the gas at t° C. and pressure p.

By Charles' Law, at the pressure p_0,

$$\text{the gas expands } \frac{v_0}{273} \text{ for } 1^\circ \text{C.}$$

$$i.e. \ \frac{v_0 t}{273} \text{ for } t^\circ \text{C.}$$

$$\therefore v' = v_0 + \frac{v_0 t}{273} = v_0\left(1 + \frac{t}{273}\right).$$

If T° C. Ab be the absolute temperature of t° C., then

$$T = t + 273.$$

$$\therefore v' = v_0\left(\frac{t + 273}{273}\right) = \frac{v_0 T}{273}.$$

By Boyle's Law a gas which has a volume of $\frac{v_0 T}{273}$ at pressure p_0 will have a volume of $\frac{p_0}{p} \cdot \frac{v_0 T}{273}$ at a pressure p, the temperature being maintained at T° C. Ab.

$$\therefore v = \frac{p_0 v_0 T}{p \cdot 273}; \quad \text{or } \frac{pv}{T} = \frac{p_0 v_0}{273}.$$

Hence, however the pressure, volume, and temperature of a given mass of gas may vary, the quantity $\frac{pv}{T}$ remains constant.

Let this constant be denoted by R.

Then $\frac{pv}{T} = R$; or $pv = RT$.

If the pressure is p' when the volume is v' and temperature T'° C. *Ab*, then $p'v' = RT'$.

$$\therefore \frac{pv}{T} = \frac{p'v'}{T'}.$$

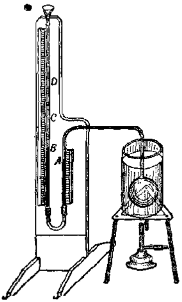
If the volume is maintained constant, so that $v' = v$, then

$$\frac{p}{T} = \frac{p'}{T'},$$

i.e. the pressure is directly proportional to the absolute temperature.

Exp.—*To prove that the pressure of a given mass of gas increases uniformly with increase of temperature when the volume is maintained constant.*

Take a glass bulb about 2 inches diameter with a long stem of narrow bore. Bend the tube twice at right angles, and join it by means of a piece of india rubber tubing to a straight piece of glass tubing about $1\frac{1}{2}$ feet long, and fix the whole on the board as in the figure.



Pour mercury into the U tube thus formed until the surface of the mercury in the arm connected to the bulb rises to a fixed mark *A*. There will thus be a mass of air enclosed in the bulb and in the part of the stem, and the pressure of this air can be found by means of the mercury gauge *ABC*.

Place the bulb in a vessel containing cold water, and adjust the mercury in the tube until the volume of the air is the same as before. Take the temperature of the water. Now find the pressure in the same manner as in proving Boyle's Law.

Now raise the temperature about 15° C., and pour mercury into the open tube until the surface in the shorter arm is at A. The fine adjustment of level can be made by raising or lowering the open tube, or by compressing the india rubber tubing. Read the temperature and take measurements for the pressure as before.

Proceed in the same manner to obtain a series of measurements of pressure at temperatures rising about 15° C.

Enter results as below.

Height of barometer = 748 mm.
Height of fixed mark A = 101 mm.

	Temperature.	Height of mercury in open branch.	Pressure.	Absolute Temperature.	Ratio of pressure to absolute temperature.
1	15·5° C.	110	757	288·5	2·623
2	30° C.	147	794	303	2·620
3	46° C.	188	835	319	2·617
4	60° C.	223	870	333	2·612
5	75° C.	263	910	348	2·615
6	88° C.	297	944	361	2·615
7	100° C.	327	974	373	2·611

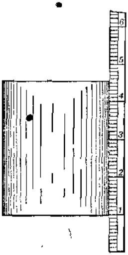
It will be seen that the ratio of the pressure to the absolute temperature is shown by this experiment to be approximately constant.

That the pressure of the enclosed gas at constant volume in*creases uniformly with increase of temperature* can be shown by plotting a curve, the ordinates of which are the increase of pressure above the pressure of the atmosphere, and the abscissae of which are the corresponding temperatures measured from 0° C. This *curve will be found very approximately to be a straight line.*

APPENDIX II.

1. *On the use of the metre scale.*

The scale should be placed with its edge along the line to be measured. The end of the scale will in time be worn away, so that the measurement should always be made beginning at the graduation marked 1. The figure will make this clear. It is possible to read to quarters of a millimetre. For example, in reading the length of the cylinder it will be seen that the end of the cylinder lies midway between the 5th and 6th millimetre division after the 4th centimetre. Hence the length is 3·55 cm. If the end of the cylinder is nearer to the 5th millimetre than the 6th, the reading may be taken to be 3·525 cm.; if, however, it is nearer to the 6th millimetre division than the 5th, the reading may be taken to be 3·575 cm.



2. *On the use of the balance.*

Observe whether the beam of the balance is horizontal. With the balance shown on page 105 this will be the case when the pointer attached to the beam is vertical, and therefore just behind the suspension-loop of the beam. If the beam is not horizontal, adjust it by placing small clips of wire or pieces of paper in one of the scale-pans.

The body to be weighed should be placed in the left-hand scale-

The box containing the weights must be placed to the right of the right-hand scale-pan.

The weights should always be put in in descending order of magnitude. The cheaper box of weights usually contains 200, 100, 50, 30, 20, 10, 5, 3, 2, 1, ·5, ·3, ·2, ·1 grammes. Suppose, therefore, *it is required to weigh a body with these weights. The operation should be performed as in the following example:*

Put in 200 gm.; it is found too little, add 100;
200 gm. + 100 gm. is found too much, take out 100 and add 50;
200 + 50 is found to be too great, take out the 50 and add 30;
200 + 30 too little, add 10;
200 + 30 + 10 too little, add 5;
200 + 30 + 10 + 5 too great, take out the 5 and add 3;
200 + 30 + 10 + 3 too little, add 1;
200 + 30 + 10 + 3 + 1 too little, add ·5;
200 + 30 + 10 + 3 + 1 + ·5 too little, add ·3;
200 + 30 + 10 + 3 + 1 + ·5 + ·3 too little, add ·2;
200 + 30 + 10 + 3 + 1 + ·5 + ·3 + ·2 too great, take out ·2 and add ·1;
200 + 30 + 10 + 3 + 1 + ·5 + ·3 + ·1 too great.
∴ the weight lies between 244·8 gm. and 244·9 gm.
The weight may be taken to be 244·85 gm.

With more delicate balance the same operation can be continued through the centigrammes and milligrammes. With more delicate balances many more precautions are necessary.

APPENDIX III.

APPARATUS REQUIRED.

	s.	d.
1. Boxwood metre scale graduated to millimetres ...	1	6
2. Scale-pans (*see* Illustration on page 105) and box of weights from 200 gm. to ·1 gm.	9	0
3. Tin vessel $7\frac{1}{2}'' \times 4''$	0	9
4. Tin vessel $4 \times 3''$	0	6
5. Tin vessel $6'' \times 1\frac{1}{2}''$	0	6
6. Iron and brass cylinder and hollow cylindrical vessel (*See* Exp. I., page 104)	6	0
7. Stands for Boyle's Law tube, and the gauges in Exp. of Chap. III.	3	0
£1	1	3

In addition there will be required the following material: Glass tubing $\frac{1}{2}''$ bore; india rubber gas tubing; rectangular pieces of deal and mahogany $1\frac{1}{2} \times \frac{3}{4}'' \times \frac{3}{4}''$; glass marbles; pieces of copper wire; lead sheet, zinc sheet or rod; sulphur; beeswax; paraffin; methylated spirits; glycerine; turpentine; paraffin oil; mercury.

The initial cost for a class of 30 boys, the boys working in pairs, will probably be £20 to £25. The wear and tear is very slight.

EXAMINATION PAPERS.

I. London Matriculation. June, 1884.

1. Distinguish between a solid and a fluid. What is the special character of a perfect fluid?

A cube of 1 foot edge is suspended in water with its upper face horizontal and at a depth of $2\frac{1}{2}$ feet below the surface. Find the pressure on each face of the cube, assuming that the mass of a cubic foot of water is 1000 oz.

2. Define specific gravity, and explain the principle of the common hydrometer. The stem of a common hydrometer is cylindrical, and the highest graduation corresponds to a specific gravity of 1·000, whilst the lowest corresponds to 1·200. What specific gravity corresponds to the point which is exactly mid-way between these two divisions?

3. What conditions limit the height to which water can be raised by means of a common pump? How must the pump be modified when it is required to raise water to a height greater than the limit thus assigned? Explain the contrivance in the lift-pump for enabling the piston-rod to work water-tight through the cylinder cover.

4. How would you determine the specific gravity of a gold medal by means of a hydrostatic balance furnished with brass weights?

Explain how each weighing and the final result will be affected by the presence of the air if no correction is made for the air

5. Describe some experimental method by which the weight of a cubic foot of air may be determined. How does the density of air vary with the pressure to which it is exposed, and how may this relation be experimentally determined?

II. London Matriculation. January, 1885.

1. A piece of iron weighing 275 grammes floats in mercury of density 13·59 with $\frac{4}{5}$ of its volume immersed. Determine the volume and density of the iron.

2. A piece of glass weighs 47 grammes in air, 22 grammes in water, and 25·8 grammes in alcohol. Find the specific gravity of the alcohol, and state the general principle on which the solution of the problem depends.

3. Describe some form of condensing air-pump. If the capacity of the barrel of the pump be 80 cc., and the capacity of the receiver 1000 cc., how many strokes will be required to raise the pressure of the air in the receiver from one atmosphere to four atmospheres?

III. London Matriculation. June, 1885.

1. A piece of silver and a piece of gold are suspended from the two ends of an equal-armed balance-beam, which is in equilibrium when the silver is immersed in alcohol (density $=0·85$), and the gold in nitric acid (density $=1·5$). The densities of the silver and gold being 10·5 and 19·3 respectively, what are their relative masses?

2. A piece of wood floats partly immersed in water, and oil is poured on the water until the wood is completely covered. Explain clearly whether this will make any change (and if so, whether there will be an increase or a decrease) in the portion of the wood below the surface of the water.

3. Give a clear explanation of the statement that the mercury in a barometer tube is sustained by atmospheric pressure. What data would you require in order to calculate the height of a glycerine barometer when the pressure of the atmosphere is 15 lb. per square

IV. London Matriculation. January, 1886.

1. Distinguish between solids and fluids, and state what you regard as the essential feature of a fluid.

A spherical boiler 4 feet in height is half full of water and half full of steam. What is the difference between the pressure at the bottom and the top of the boiler?

2. Distinguish between the absolute and apparent weight of a body. The apparent weight of a piece of platinum in water is 60 grammes, and the absolute weight of another piece of platinum twice as big as the former is 126 grammes. Determine the specific gravity of platinum.

3. Glycerine rises in a barometer tube to a height of 26 feet when mercury stands at 30 inches. The specific gravity of mercury is 13·6. Find that of glycerine.

If an iron bullet were allowed to float on the mercury in a barometer tube, how would the height of the mercury be affected?

4. State Boyle's Law.

If a quantity of air be saturated with aqueous vapour, will it obey Boyle's Law (*a*) when compressed, (*b*) when allowed to expand? and if not, in what way will it diverge from the law?

V. London Matriculation. June, 1887.

1. Explain what is meant by the statement that the specific gravity of mercury is 13·596. The lower ends of two vertical tubes, whose cross sections are 1 and ·1 square inches respectively, are connected by a tube. The tubes contain mercury. How much water must be poured into the larger tube to raise the level of the mercury in the smaller tube by one inch?

2. A glass tapering tube is 10 inches long, 1 inch diameter at one end, and half-an-inch at the other. The tube is held vertical and filled with water, a thin plate (the weight of which may be neglected) being pressed against the lower end to prevent the water from escaping. Compare the forces with which the plate must be

held in its place, according as the larger end of the tube is up or down.

3. Describe a siphon, and explain its action, showing clearly why it is necessary that the end out of which the liquid flows should be below the level of the liquid in the vessel from which it is drawn.

VI. London Matriculation. January, 1888.

1. Explain the principle of a siphon. What would happen if a small hole were made (*a*) in the shorter limb, (*b*) in the longer limb of a siphon in action?

2. Define density. The specific gravity of brass referred to water is 8. Taking the mass of one cubic foot of water as 1000 oz., find the density of brass in ounces to the cubic inch.

3. How would you prove experimentally that when a body is immersed in a liquid its loss of weight is equal to the weight of the liquid displaced?

A ship weighing 1000 tons goes from fresh water to salt water. If the area of a section of the ship at the water line be 15,000 square feet, and her sides vertical where they cut the water, find how much she will rise, taking the specific gravity of sea-water as 1·026.

4. A piece of wax weighs $4\frac{1}{2}$ grammes in air. A piece of platinum, whose volume is one-tenth of a cubic centimetre and specific gravity 21, is attached to the wax, and the two together weigh in water $1\frac{1}{2}$ grammes. Find the specific gravity of the wax.

VII. London Matriculation. January and June, 1889.

1. How would you show experimentally that the difference in pressure at two points in a heavy liquid is proportional to the difference in depth of the points?

2. With the barometer at 760 mm., the mass of a litre of air is 1·2 grammes, and of a litre of hydrogen ·089 grammes. The material of a balloon weighs 50 kilogrammes. What must be its

volume in order that it may just rise when filled with hydrogen? Explain carefully how you obtain your result.

3. Describe some method of finding the specific gravity of a fluid.

A certain body just floats in water. On placing it in sulphuric acid of specific gravity 1·85 it requires an addition of 42·5 grammes to immerse it. Find its volume.

4. A cylinder whose base is a circle 1 foot in diameter, and whose height is 3 feet, weighs 10 lb. Calculate its density, assuming 1 cubic foot of water to weigh 62·5 lb.

5. Describe experiments to prove that if the temperature remain constant the pressure of a gas is proportional to its density.

6. Describe carefully the construction and mode of action of a mercurial barometer. Does the height of the barometer depend on the diameter of the tube? Give reasons for your answer.

VIII. London Matriculation. January and June, 1890.

1. Show how to calculate the force exerted by a fluid on a plane surface in contact with it.

2. Define specific gravity, and show how to find it for a solid by means of a specific gravity bottle. Explain why the result of the experiment is specific gravity as defined.

3. Explain why the barometer goes down when carried up a mountain. When carried up 1500 feet will it go down 15 times as much as it does when carried up 100 feet? Give reasons for your answers.

4. Prove that the surface of a heavy liquid at rest in a vessel is a horizontal plane.

5. The upper ends of two long vertical glass tubes are connected together and to the receiver of an air-pump. The lower end of one dips into a beaker of water, that of the other into a beaker of copper sulphate. On working the air-pump, the liquids rise in the two [illegible]

8. A diving-bell, having a capacity of 125 cubic feet, is sunk in salt water to a depth of 100 feet. If the specific gravity of salt water be 1·02, and the height of the water barometer be 34 feet, find the total quantity of air at atmospheric pressure required to fill the bell.

IX. London Matriculation. January, 1891.

1. Define density or specific gravity, and calculate the mass of one cubic centimetre of a certain solid from the following data: A mass of 720 grammes hanging from one pan of a balance is totally immersed in water, and found to be counterpoised by a weight of 645 grammes in the other pan.

2. What do you know about the density of gases in relation to temperature and pressure? Describe experiments which show that the density of a gas at constant temperature is proportional to its pressure. A uniform tube closed at top, open at bottom, is plunged into mercury, so that it contains 25 cc. of gas at atmospheric pressure of 76 cm.; the tube is now raised until the gas occupies 50 cc. How much has it been raised?

3. Explain the construction of a barometer; what it measures, and how it measures it. Translate pressure measured in terms of the height of a barometer mercury column (say either 27 inches or 60 centimetres) into absolute units of pressure.

X. Cambridge Local Examinations. Dec. 1887, 1888.

Junior Candidates.

1. A water bottle has a broad base and slides sloping up to a narrow neck. Show that when the bottle is full of water the force exerted by the water upon the base is greater than the whole weight of the water in the bottle.

2. Under what circumstances will a body float in a liquid? A block of iron floats in mercury. Water is poured on the top of the mercury so as to cover the iron by a depth of several inches of water. Will the iron rise or sink in consequence? Give reasons for your answer.

3. Describe a spirit level, and explain its use.

Show that a plane board can be set level if it be provided with three levelling screws. How would you proceed to set it level?

4. Enunciate Boyle's Law, and describe any simple experiment by which it may be verified.

A tube which is drawn to a point at one end has its pointed end dipped into water, and a finger is firmly pressed against the open end of the tube. Show that a considerable portion of the water in the lower part of the tube may be lifted out without spilling.

5. Explain the use of the safety valve of a steam engine.

A safety valve of a boiler consists of a square iron plate hinged along one side, and a weight of 700 lb. is attached to that side which is furthest from the hinge. If the weight of the iron plate be 40 lb., and its area one square foot, find the greatest pressure of the steam which can be maintained in the boiler.

I. Cambridge Local Examinations. Dec. 1887, 1888.

Senior Candidates.

1. Define a solid and a fluid.

Express the pressure of the atmosphere in lbwt. per square inch if it will support a column of water 32 feet high. (The density of water is 62·3 lb. per cubic foot.)

2. Explain what is meant by the density of a substance.

A beaker of water is placed in a scale-pan of a balance and counterpoised; a mass of copper weighing 34 grammes is suspended by a fine wire from an independent support so as to be completely immersed in the water in the beaker; the density of copper is 8·9 grammes per cc. Find the volume of the copper and the alteration that must be made in the counterpoise so that the beaker may be balanced when the copper is immersed.

3. How can the specific gravity of a powder be obtained? A solid whose specific gravity is 1·8 is weighed in a mixture of alcohol (sp. gr. = ·8) and water. If the solid weighs 72 ounces in the air and 35 ounces in the mixture, find the proportion of the alcohol to

4. Show that the resultant pressure on a body completely immersed in a liquid is such as would sustain a mass of liquid whose volume is equal to that of the immersed body.

A closed tube contains water in which a piece of cork is floating, the water and the cork together completely filling the tube. The tube is fixed at right angles to a vertical axis through its centre, about which it is made to rotate. Will the cork remain near the axis of rotation, or fly out towards one end of the tube?

XII. Cambridge Local Examinations. Dec. 1887, 1888.

SENIOR CANDIDATES.

1. "A fluid is a substance in which any distribution of forces tending to make one part slide over the other will always produce a relative displacement of those parts which continually increases with the continued action of the forces."

"A fluid is a substance such that the action between two contiguous parts separated by a plane surface is a pair of equal and opposite forces perpendicular to the plane surface."

Reconcile these two definitions. Is there any substance which is accurately described by either or both of these definitions?

2. The density of a substance being defined as the mass of a unit volume of the substance, explain how the density of water may be practically determined.

The height of a glycerine barometer on May 7th was $27\frac{1}{4}$ feet, the height of the mercury barometer being 30·4 inches at the same time. Find the specific gravity of glycerine, that of mercury being 13·56.

3. What are the conditions of equilibrium of a floating body?

A rectangular block of wood two feet long by one foot wide and one deep is put into a rectangular vessel, the length and breadth of which exceeds those of the block by half-an-inch each. If the weight of the wood is 100 lb., find the weight of the water which must be poured into the vessel in order to float the block, a cubic

4. Show that the pressure at any point in a heavy liquid at rest is proportional to the depth below the free surface of the liquid.

A barometer tube stands in a mercury trough. At what point will the pressure be most likely to crush the tube?

XIII. Cambridge Local Examinations. Dec. 1890, 1891.

JUNIOR CANDIDATES.

1. *Distinguish between density and specific gravity.* What is the relation between the numbers representing the density and the specific gravity of any given substance in the metric system?

Equal weights of two liquids are mixed whose densities are in the ratio 3 : 2. The volume of the mixture shrinks to four-fifths of that of the sum of its components. Find its specific gravity in terms of either.

2. *The surface of a liquid at rest is horizontal. Prove this.* State the conditions of equilibrium of a body floating in a liquid, and give the magnitude and direction of the resultant pressure.

3. Explain how an iron ship can float in the water. What advantage is gained by building the ship in compartments?

If the attraction of the earth were doubled, would the ship rise or sink in the water?

4. Explain any method of comparing the specific gravity of two different liquids.

A block of wood floats in a liquid with four-fifths of its volume immersed. In another liquid it floats with two-thirds of its volume immersed. If the liquids are mixed together in equal parts by weight, what fraction of the volume of the wood will be immersed?

5. How does the volume of a gas vary according to its pressure?

The weight of a diving-bell appears to increase as it is lowered in the water. Explain this. Find approximately the apparent increase of weight at a depth of 100 feet if the capacity of the bell is 1000 cubic feet.

6. Describe the diving-bell, and explain its use.

A large wooden ball floats in the water inside a diving-bell. Will the ball appear to a person seated inside the bell to rise or sink in the water as the bell is lowered?

XIV. Cambridge Local Examinations. Dec. 1890, 1891.

SENIOR CANDIDATES.

1. What is meant by the specific gravity of a substance?

Describe two different methods by which it may be found in the case of a liquid. How would you find the specific gravity of sand?

2. What are the conditions of equilibrium of a body floating in a liquid?

A cubical box of 1 foot external dimensions, made of material of thickness 1 inch, floats in water immersed to a depth of $3\frac{3}{4}$ inches. How many cubic inches of water must be poured in so that the water outside and inside may stand at the same level? And how deep in the water will the box then be?

3. Explain the construction and action of the ordinary mercurial barometer.

A long uniform tube closed at one end is nearly filled with mercury, and inverted in a deep well of the same liquid. The mercury stands in the tube at 20 inches above the surface of the mercury in the well, with 10 inches above it occupied by air. The tube is then cautiously lifted until the air occupies 20 inches of the tube, when the mercury is observed to stand at $24\frac{1}{2}$ inches. Find how much of the tube the enclosed air would fill at the atmospheric pressure, and find the atmospheric pressure.

4. State and illustrate Boyle's Law.

A uniform bent tube with one end closed is held upright, and mercury is poured into the open arms till it reaches the same level in the two arms. The length of the closed tube above the mercury being 15 inches, and the height of the barometer 30 inches, how much more of the open tube must be filled with mercury in order that the air may occupy only 10 inches in the other?

XV. Cambridge Local Examinations. Dec. 1890, 1891.

SENIOR CANDIDATES.

1. What is meant by the resultant pressure of a fluid on a body immersed in it? Find the magnitude and direction of this pressure when the body is immersed in water.

2. What are the uses of a hydrometer? Describe some form of the instrument.

A Nicholson's hydrometer has a small pan at the top of its stem, and is observed to float at a certain level when weights equal to 10·25 grammes are in this pan. The weights being replaced by a small substance, it floats again at the same level; and when the substance is tied to the instrument below water, and 7·75 grammes are placed in the pan, it again floats at the same level as before. Find the specific gravity of the substance.

3. Enunciate and explain Boyle's Law.

A pint bottle, containing atmospheric air, just floats in water when weighted with 5 ounces. The weight is then removed, and the bottle is immersed neck downwards and gently pressed down. Show that it will just float freely when the level of the water inside the bottle is 11 feet below the surface, will sink if lowered further, and will rise if raised higher. (The water barometer stands at 33 feet, and a pint of water weighs 20 ounces.)

4. What determines the lifting power of a balloon?

Prove that if a partially-filled balloon is allowed to ascend, the only way to stop its motion before it is fully expanded is to let out gas; and show that wherever it comes to rest the same amount of gas will always be left in the balloon.

XVI. Oxford Local Examinations. June, 1887, 1888.

JUNIOR CANDIDATES.

1. Describe and explain the action of the common pump. From how deep a well can such a pump at the surface of the ground raise

2. A tank in the form of a cube whose edge is 1 foot is full of water, 1000 ounces of which just fill it. Find in pounds the pressure on each of its faces.

If half of the water be drawn out and replaced by oil of specific gravity ·75, find the new pressure on each face.

3. Explain the term *elastic fluid*.

The tube of a barometer rises to 33 inches above the level of the mercury in the trough from which it rises. Also the height of the mercurial column in it is 27 inches when it should be 30 inches, thus indicating that there is air in the tube. Prove that this air, if at atmospheric pressure, would occupy three-fifths of an inch of the tube.

4. Two liquids which do not mix meet in a bent tube. Show how to compare their specific gravities from observation of the heights of the three surfaces.

5. Find the whole pressure, in pounds weight, on one vertical side of a cubical tank, 10 feet deep, full of water, the weight of one cubic foot of water being 1000 oz.

6. Describe the single-barrelled air-pump, and explain its working.

If the volume of the cylinder be equal to that of the receiver, what portion of the air can be driven out in 3 strokes?

XVII. Oxford Local Examinations. June, 1887, 1888.

Junior Candidates.

1. How can it be experimentally verified that fluids press equally in all directions?

Describe Bramah's press, explaining its use and stating clearly the principle on which its efficiency depends.

2. If a body float in water, prove that its weight is equal to that of the water displaced.

How much water will run over the edges of a cup just full of water when a cork of volume 2 cubic inches and specific gravity ·2 is placed gently so as to float in it?

3. What is the law connecting pressure and volume of a given quantity of gas at a given temperature?

A cylindrical diving-bell 10 feet high is sunk to the bottom of a river on a day when the atmospheric pressure at the surface is that produced by a column of water 32 feet high, and the water is found to rise 2 feet in the bell. Find the depth of the river.

4. A body appeared to weigh 5 lb. when weighed in water, and $5\frac{1}{2}$ lb. when weighed in oil of specific gravity ·75. Find its specific gravity and its true weight.

5. State the law connecting the pressure and density of a gas when the temperature is constant.

A ball one foot in diameter, containing air at atmospheric pressure, is compressed until it is 6 inches in diameter, without the air escaping. Compare the pressure of the air within it with that of the atmosphere.

6. Describe the condenser, and explain its working.

If the volume of the cylinder be half that of the receiver, in how many strokes will the pressure in the receiver be doubled?

XVIII. Oxford Local Examinations. June, 1887.

Senior Candidates.

1. A vessel of water is placed in one scale of a balance, and there is a weight in the other which will just counterbalance it. Will the equilibrium be disturbed if a person dips his finger in the water without touching the sides of the vessel? Give reasons for your answer.

2. A rectangular plate measuring 8 inches by 12 inches is immersed vertically in mercury with one of its *shorter sides in the surface*. Assuming that two cubic inches of mercury weigh 1 lb., and that there is no pressure on the upper surface of the mercury, find (*a*) the pressure at a point of the plate 8 inches below the surface, (*b*) the whole pressure on one side of the plate.

3. Explain how a thermometer is graduated, and show how to compare the scales of two differently graduated thermometers.

The number which expresses a certain temperature on the Réaumur scale is equal to the sum of the numbers which express the same temperature on the Fahrenheit and Centigrade scales respectively. Find the numbers.

4. Sketch the experiment of Torricelli whereby he estimated the pressure of the atmosphere.

What is the mean pressure of the atmosphere in pounds per square inch and grammes per square centimetre? (Specific gravity of mercury = 13·6.)

5. Describe (*a*) a Sprengel mercury pump, (*b*) a manometer.

XIX. Oxford Local Examinations. June, 1887, 1888.

Senior Candidates.

1. Prove that the surface of a liquid at rest under the action of gravity is a horizontal plane.

A parallelogram in any position is totally immersed in water. Prove that the sum of the pressures at the extremities of one diagonal is equal to the sum of the pressures at the extremities of the other diagonal.

2. Describe the common barometer.

If a barometer with an imperfect vacuum stand at heights of 29 and $29\frac{1}{4}$ inches when the true heights are 30 and $30\frac{3}{8}$, what will be the true height when it stands at $29\frac{1}{8}$ inches?

3. Describe Smeaton's air-pump, and find the density of the air in the receiver after n ascents of the piston.

4. How can the specific gravity of a liquid be found?

5. Define the whole pressure and the resultant pressure of a fluid on a surface.

A closed cubical box 12 inches high is not quite full of mercury of specific gravity 13·6, and is placed on the flat bottom of a pool of water. What must be the depth of the pool in order that the whole pressure from within on one of the vertical faces of the box

6. State the relation between the pressure and volume of a given quantity of air at a given temperature.

Two spheres, whose radii are 1 and 2 feet respectively, contain the same mass of air at the same temperature, and the pressure a any point in the first sphere is 16 lb. per square inch. Find th pressure on the other sphere.

7. Describe the construction and use of the common hydrometer

If it floats in water with one inch of the stem above the surface and in liquid of specific gravity 1·2 with 3 inches above, wha will be the specific gravity of a liquid in which it floats with 5 inches above?

8. Describe the Bramah press, pointing out how, in its action the law of conservation of energy is obeyed.

XX. Oxford Local Examinations. January, 1890, 1891

JUNIOR CANDIDATES.

1. Define density.

Two liquids are such that when they are mixed in volumes i the proportion of 4 : 6, the density of the mixture is the same a that of water; but if to any volume of the mixture there is adde one-fourth as much of the lighter liquid, the density of the mixtur will be reduced to ·95. Find the densities of the two liquids.

2. Define the pressure at any point of a fluid. Find the pres sure per square inch on the base of a cone, the height of which is 1: inches, standing with its vertex upwards and filled with water. (A cubic foot of water weighs 1000 oz.)

3. Prove that the pressure at any point in a liquid at rest i proportional to the depth below the surface.

How deep must a 3-inch cube be sunk in water with two of it faces horizontal, so that the whole pressure on 5 square inches o the bottom may be equal to that on 6 square inches of the top. And what will then be the whole pressure on each of the sides?

4. What are the conditions of equilibrium of a body floating i

A cubical box floats in water half immersed. When a weight of 3 lb. is put into it, two-thirds of it are immersed. Find the weight of the box.

5. A mercurial barometer is sunk in a vessel of water, and reads 32·18 inches when its free surface is three feet below the surface of the water, while outside the reading is 29·3 inches. Compare the densities of mercury and water.

6. If a cylindrical diving-bell is let down when the mercurial barometer stands at 30·1 inches and the water barometer at 32·2 feet until a mercurial barometer in the bell indicates 34·4 inches, what is the depth of the water in the bell below the surface of the water outside?

XXI. Oxford Local Examinations. June, 1890, 1891.

Senior Candidates.

1. The interior of a vessel, in form a hollow pyramid 4 inches high on a square base, each side of which is 6 inches, is filled with water. Describe the forces acting on the water which keep it in equilibrium. Also determine the whole pressure exerted by the water on the base, and on each of the faces of the pyramid. (*A cubic foot of water weighs 1000 oz.*)

2. *Prove that when a body floats in equilibrium, the weight of* the liquid displaced is equal to that of the body.

If a conical gallon measure floats in water vertex downwards, with three-fifths of its axis below the surface of the water, how much water poured in will immerse the measure up to the brim?

3. Show that a floating body is in stable or unstable equilibrium according as the metacentre is above or below the centre of gravity of the body.

A uniform bar whose specific gravity is ·945, moveable freely about a fixed point at one of its ends, which is above the surface of a liquid of specific gravity 1·445, rests obliquely and is partly immersed in the liquid. Determine what proportion of the bar will be above the liquid.

4. Explain the construction of a hydrostatic balance.

A 3-inch cube of one substance weighs 14·75 oz. in air, and a 4-inch cube of another substance weighs 62·5 oz. in water. Find the specific gravity of each substance, and the mean specific gravity of the two cubes fastened together, a cubic foot of water being assumed to weigh 1000 oz.

5. Describe a forcing pump, and explain its action by the aid of a diagram.

6. Explain the principle of the action of a siphon, and determine the limit of its use with olive oil when the barometric height is 76·25 cm., the densities of mercury and olive oil being 13·596 and ·915 respectively.

XXII. Cambridge First M.B. Examination. Dec. 1887.

1. A U tube open at each end is partially filled with mercury, and water is then placed in one leg. Find what relation exists between the levels of the liquids in the two legs. If the mercury stands half-way up the legs of the U tube before the water is poured in, and if one leg be then filled to the top with water, find the height of the mercury in the other. The specific gravity of mercury is 13·6.

2. Define density. Describe a method of determining the density of a solid lighter than water.

A solid has a weight of 125 grammes in air, 75 grammes in water, and 60 grammes in a solution of salt and water. Find the density of the solid and of the solution.

3. State Boyle's law for the pressure of a gas.

Two cubic centimetres of air are measured off at atmospheric pressure. When introduced into the vacuum of a barometer they depress the mercury which previously stood at 76 centimetres and occupy a volume of 15 cubic centimetres. By how much has the mercurial column been depressed?

4. Define the pressure at a point in a fluid, and show that the pressure at a point in a heavy fluid is proportional to the depth of the point.

The lower portion of a U tube contains mercury. How many inches of water must be poured into one leg of the tube to raise the mercury 1 inch in the other, assuming the specific gravity of mercury to be 13·6?

5. How would you show experimentally that the pressure on a body immersed in a fluid is equal to the weight of the fluid displaced?

6. Describe the construction and mode of action of an air-pump. The volume of the receiver of an air-pump is 4 times that of the barrel. Show that after 5 strokes the air will be reduced to less than one-third of its initial density.

7. Describe the construction of an ordinary barometer designed to give accurate readings.

A barometer has some air at the top. How would this affect the reading (*a*) on taking the barometer down a coal-pit, (*b*) on raising the temperature?

XXIII. Cambridge First M.B. Examination. Dec. 1888.

1. What conditions determine whether a body shall float or not?

A loaded piece of cork and an elastic bladder containing air just float at the surface of the sea. What will happen if they are both plunged to a great depth and then released?

2. Describe a method of finding the specific gravity of a solid soluble in water.

A specific gravity bottle full of water weighs 50·3 grammes. Some pieces of metal weighing 17·6 grammes are introduced and the bottle is again filled with water, the combined weight being 65·9 grammes. Find the specific gravity of the metal.

3. Explain the principle of the common hydrometer.

The density corresponding to the lowest mark on the stem of a certain hydrometer is 1, while that corresponding to the highest is ·8. Find the specific gravity corresponding to a point midway

4. How would you prove Boyle's law for pressures less than one atmosphere?

The density of mercury is 13·6, and the barometer stands at 30 inches. A bubble of gas, the volume of which is 1 cubic inch at the bottom of a lake 170 feet deep, rises to the surface. What will its volume be when it reaches the surface?

XXIV. Cambridge First M.B. Examination. June, 1888.

1. State the laws of Boyle and Gay-Lussac.

A litre of air at 0° C., and under atmospheric pressure, weighs 1·2 grammes. Find the mass of the air required at 18° C. to produce a pressure of 3 atmospheres in a volume of 75 cubic centimetres.

2. A cubic metre of wood floats in water with three fourths of its volume immersed. Calculate the depth to which it will sink in a liquid of specific gravity ·8.

3. Describe the siphon and its action.

What would be the effect of having a large bubble of air at the highest point of the siphon tube?

4. A tumbler is pushed mouth downwards under water until it is half filled with water. Find the depth of the tumbler below the surface of the water.

XXV. Cambridge First M.B. Examination. June, 1889.

1. Describe Nicholson's hydrometer.

If the sinking weight is 20 grammes, find the specific gravity of a solid such that the weights required to bring the instrument to the standard position are 11 grammes when it is in the upper pan and 14 grammes when it is in the lower pan.

2. A cylindrical cork 4 inches long is to be loaded with iron of the same section as itself so as just to float in water. How long must the iron load be?

With what acceleration would the unloaded cork move if it were plunged to a depth below the surface of the water and then released?

3. A spring balance is graduated by means of experiments performed in a vacuum. In what way will the weight of heavy objects such as gold, as given by this balance in air, differ from the true weight? What corrections will have to be made?

How does the presence of the air affect the weight of such objects as given by an ordinary balance?

4. State Boyle's law for the pressure of gases. A barometer is filled with mercury before inverting up to within one inch of the top. After inversion the air expands till it occupies 15 inches of the tube, and the mercury stands 28 inches in the tube above the level of the mercury in the trough. Find the true height of the barometer at the time.

XXVI. Cambridge First M.B. Examination. June, 1890.

1. The lower portion of a U tube with vertical branches contains mercury (specific gravity 13·6). Some liquid is poured into the right hand branch till it occupies 12 inches of the tube. The difference of level of the mercury on the two sides is found to be 4 inches. What is the specific gravity of the liquid?

2. A large stone is held suspended under water by a rope. Explain why the load on the rope is less under these conditions than if the stone were suspended in the air. If an addition of 10 lb. to the load on the rope will break the rope, how much of the stone may be raised out of the water before the rope breaks? (A cubic foot of water weighs $62\frac{1}{2}$ lb.)

3. Explain the principle of the common hydrometer.

The specific gravity corresponding to the lowest mark on the stem of a certain hydrometer is 1·8. What must be that corresponding to the highest mark if the reading midway between the two is 1·6?

4. State Boyle's law relating to gases.

A certain bottle, which has an outlet at the bottom fitted with a tap, is nearly filled with water, and is closed with an air-tight stopper. When the tap is turned on, the water runs out, at first

quickly, then more and more slowly, and at last not at all, though there *still remains* water in the bottle. *Explain this*, and state what difference of pressure exists between the air in the bottle and that outside when the flow has ceased.

XXVII. Cambridge First M.B. Examination. June, 1891.

1. *State how the pressure varies at different depths below the surface of water.*

Find the difference of the pressure tending to distend the blood-vessels in the brain according as a man 6 feet high stands (a) on his head, (b) on his heels. What would be the difference in pressure if he performed the same feat under water? You may assume that the specific gravity of blood is the same as that of water.

2. A beaker half full of water and a thimble are put into one pan of a balance and weighed. What, if any, differences in the weight will be observed (a) if the thimble is made to float on the water in the beaker, (b) if the thimble is then allowed to fill itself with water and sink to the bottom of the beaker? Give your reasons in each case.

3. A lump of metal is known to consist of silver and gold, but *it is not known how much is gold and how much silver.* The metal weighs 20 grams in air and 18·7 in water. How much gold is there in the mixture? (Specific gravity of gold = 19·3, that of silver = 10·5.)

4. Explain how you would determine by experiment the pressure of the gas contained in a given large reservoir.

XXVIII. General Examination for the Ordinary B.A. Degree. June, 1887.

1. Distinguish between a solid, a liquid, and a gas.

What is meant by a perfect fluid?

2. Explain how fluid pressure is measured.

If the atmospheric pressure be 14 lb. on the square inch, h

3. Prove that the pressure at any depth below the surface of a heavy incompressible fluid is proportional to the depth below the surface.

A vessel whose bottom is horizontal contains mercury whose depth is 20 inches, and water floats on the mercury to the depth of 16 inches. Find the pressure at a point on the bottom of the vessel in lb. on the square inch, the specific gravity of mercury being 13·6 and the weight of a cubic foot of water 1000 ounces.

4. Find the resultant and line of action of the resultant of the fluid pressures upon a body wholly or partially immersed in a fluid. Find the conditions of equilibrium of a body floating in a fluid. How must the conditions be modified if a body float immersed in two or more fluids of different specific gravities?

A body floats in water contained in a vessel placed under an exhausted receiver with half its volume immersed. Air is then forced into the receiver till its density is 80 times that of air at atmospheric pressure. Show that the volume immersed in water will then be four-ninths of the whole volume, assuming the specific gravity of air at atmospheric pressure to be ·00125.

5. Define the specific gravity of a body.

Describe Nicholson's hydrometer, and explain how it is used (*a*) to find the specific gravity of a solid heavier than water; (*b*) to find the specific gravity of a liquid.

6. Describe the siphon, and explain its action. How is the action of the siphon affected by a change in the atmospheric pressure?

XXIX. General Examination for the Ordinary B.A. Degree. June, 1888.

1. Define a fluid, and state the property which is assumed as the basis of all reasoning upon fluid action.

Prove that the pressure at any point in an incompressible fluid varies as the depth below the surface, the pressure at the surface being

2. Define a horizontal line, and show that the surface of liquid at rest is a horizontal plane.

3. What are the conditions of equilibrium of a body floating in liquid and only partly immersed?

A body whose density is 3 and whose weight is 6 lb. is supported by a string with half its volume immersed in water. What is the tension of the string, the density of water being unity?

4. Prove that the pressure of a liquid on any plane area immersed in it is equal to the weight of a column of the liquid of which the area is the base and of which the length is equal to the depth of the centre of gravity of the area below the surface of the liquid.

5. Show how to compare the specific gravities of a solid and a liquid by means of the hydrostatic balance when the solid is heavier than the liquid.

A solid of one ounce rests at the bottom of a vessel of water. If the pressure of the solid on the bottom of the vessel be [illegible] oz., find its specific gravity, water being the standard.

6. Describe Smeaton's air-pump, and show how to find the density of the air in the receiver after n ascents of the piston.

Find the ratio of the volume of the receiver to that of the barrel, if at the end of the fourth stroke the density of the air in the receiver is to its original density as 256 to 625.

XXX. General Examination for the Ordinary B.A. Degree. December, 1890.

1. Define a liquid, and distinguish it from a gas. How does the ideal fluid differ from fluids which we find in nature?

How is the pressure at a point in a fluid measured?

2. Prove that the surface of a heavy fluid at rest is horizontal.

To what depth must a surface 1 square inch in area be sunk in the sea that the pressure on one side of it may be ·9125 of a ton? (Take a cubic foot of sea-water to weigh 1022 ounces.)

3. *Find the resultant of the fluid pressures on a body completely immersed in a fluid.* What is the object in supposing the fluid to become solidified during the argument?

What (*a*) volume, (*b*) weight of water will run over from a full vessel of water when a pound of cork is gently placed in it?

4. How may the specific gravity of a substance lighter than water be measured?

A bladder filled with 12 cubic feet of air floats in water. How many pounds of lead (sp. gr. 11·3) will be required to sink the bladder?

5. Describe some form of forcing pump.

What is the advantage of it as compared with other pumps?

6. Describe the construction of the mercurial barometer.

If the tube is 35½ inches long, and there is a little air at the top, making the reading 28½ inches when it ought to be 30¾ inches, show that the reading will be 26½ inches when it should be 28¼ inches.

XXXI. General Examination for the Ordinary B.A. Degree. June, 1891.

1. What are the two great classes into which fluids are divided? By what properties are these distinguished from each other? What is meant by the pressure of a fluid at a given point?

State the hydrostatic paradox, and show how it is illustrated by the hydrostatic bellows.

2. Prove that in a liquid at rest the pressure is the same at all points in the same horizontal plane.

How is this proposition proved when the line joining the two points does not lie entirely in the fluid?

3. A body floats freely in water, partially submerged. Find the conditions of equilibrium.

A straight uniform rod, whose specific gravity (referred to water) is [illegible], has one end attached to a string, whose other end is attached,

to a point vertically above a tank of water whose surface the string is not quite long enough to reach. When the rod is in equilibrium, find what fraction of its length is submerged.

4. Define the specific gravity of a body.

Show how to compare the specific gravity of two liquids by means of the hydrostatic balance.

A piece of brick, which weighs 10 ounces in air, weighs 5 ounces in water and 6 ounces in spirit. Find the specific gravity of the spirit compared to water.

5. State the law connecting the pressure of air at a given temperature with its density.

When the barometer is standing at 30 inches, a uniform straight tube, closed at one end, is partly filled with mercury and held in a vertical position with its closed end upwards and with its open end in a tank of mercury. When the closed end is 25 inches above the surface of the mercury in the tank, the surface of the mercury inside the tube is one inch from the closed end. How far must the tube be pulled up in order that the top of the mercury inside it may be 6 inches from the closed end?

6. Describe the common pump, and explain its action.

What would be the difference in the effect produced by a small hole, according as it was above or below the fixed valve?

XXXII. Science and Art Examinations. 1885.

(E) = Elementary. (A) = Advanced. (H) = Honours.

1. A cube, each of whose edges is 2 feet long, stands on one face on the bottom of a vessel containing water 4 feet deep. Find the pressure of the water on one of the upright faces of the cube, assuming that a cubic foot of water weighs 1000 oz. (E)

2. Define the specific gravity or specific density of a solid or liquid. Given the specific gravity of petroleum is 0·88, and that a quart of water weighs 40 oz., find how many gallons of petroleum

3. Describe briefly and explain the action of a common fo pump. When the piston (or plunger) is lifted, why does wa follow it? If the plunger has a cross section of 4 square incl and works 80 feet below the cistern, with what pressure must it pushed down in order to force water into the cistern? (E)

4. A sphere lighter than an equal volume of water has its cen of gravity not coincident with its geometrical centre. State in wl position it will float if put in water, and explain why it takes tl position. (A)

5. Explain the action of the siphon. A vessel containing wa is being emptied by means of a siphon. What effect would be p duced by making a small hole through the top of the siphon? (

6. Find the pressure of a liquid on a body wholly or par immersed in it. A right prism has a rectangular base $ABCD$; specific gravity is 0·5; it floats in water with its axis horizontal; is displaced so that the diagonal BD is on the surface of the wat Find the moment of the couple tending to bring it back to original position of equilibrium. (H)

7. Find the centre of pressure of a circular area just immers in water with its plane vertical. (H)

8. How is the pressure of a liquid on the base of a vessel asc tained? A rectangular tank has an inclined base $ABCD$, where the side AB is 4 feet long, and each point of it is 4 feet below t surface of the water, while CD is also 4 feet long, each point bei 2 feet below the surface of the water. Find the pressure in poun upon the base, it being given that $AD = BC = 6$ feet, and al that a cubic foot of water weighs $62\frac{1}{2}$ lb. (E)

9. Sketch sectional elevation of hydraulic press, showing mo of packing ram. The power which actuates a force pump is appli at the end of a lever giving a mechanical advantage of 14 to 1, th area of plunger of pump is 1 square inch. What power must l applied to the end of the lever to produce a pressure of 1 ton p

XXXIII. Science and Art Examinations. 1886.

1. When a body is wholly or partly immersed in a liquid, what is the magnitude of the resultant pressure of the liquid on the body? A body whose specific gravity is 1·4 and *volume* 3 cubic feet is placed in a vessel in which there is water enough to cover it. What pressure does the body produce on the points of the bottom at which it is *supported?* (E)

2. Explain briefly the method for finding the specific gravity of an insoluble body by *means of* the balance. If a body weighs 732 grains in vacuo and 252 grains in water, what is its specific gravity? (E)

3. A tube filled with water is inverted with its open end in water, no air having got in; the top of the tube is 20 feet above the surface of the external water. If the water barometer stands at 34 feet, what is the pressure in pounds per square foot at a point of the inside of the top of the tube? (1 cubic foot of water weighs 1000 oz.) What would be the consequence of making a hole in the top of the tube, and why would the consequence follow? (E)

4. Define the centre of pressure of a fluid on a plane area, and find its position in the case of a rectangular area, one edge being on the surface of the fluid. Find where the centre of pressure of the rectangle would be if its plane were vertical and its upper edge (which is horizontal) below the surface of the water at a distance equal to the height of the rectangle. (A)

5. Given the specific gravities of two liquids, show how to calculate the specific gravity of a mixture of given volume of the two liquids, assuming that the mixture takes place without a change of volume. Three pints of a liquid whose specific gravity is 0·8 are mixed with five pints of another liquid whose specific gravity is 1·04. Find the specific gravity of the mixture (a) if there is no contraction, (b) if on mixture there is a contraction of 5 per cent. of the joint volumes. (A)

6. On a vertical wall of a reservoir a circle is described with its centre on the surface of the water. Find the centre of fluid pressure

7. In the vertical plane side of a tank holding water there is a rectangular plate whose depth is 1 foot and breadth 2 feet, the upper edge being horizontal and 8 feet below the surface of the water. Find the pressure on the plate. (The weight of a cubic foot of water is $65\frac{1}{2}$ lb.) (E)

8. Sketch a section through a hydraulic press, and describe its action by reference to your drawing. Give the diameters of the plunger and ram which you would adopt in order that a load of 2 tons on the ram may be raised by a pressure of 20 lb. at the end of the lever handle which works the force pump and acts with a leverage of 8 to 1. (E)

XXXIV. Science and Art Examinations. 1887.

1. When a body is wholly or partly immersed in water, what is the magnitude of the resultant fluid pressure on it? In what direction and along what line does the resultant act? A body whose volume is 5 cubic feet and specific gravity 1·2 hangs by a string suspended in water which completely covers it. What force exerted along the string is needed to support the body? (E)

2. Describe Nicholson's hydrometer, and how it is used for finding the weight of a small body. The standard weight being 1200 grains, a body is placed in the upper pan, and it is found that 200 grains must be added to sink the hydrometer to the standard point. The body is now placed in the lower pan, and it is found that 450 grains must be placed in the lower pan to sink the instrument to the standard point. What is the specific gravity of the body? (E)

3. If two liquids do not mix, show that their common surface is a horizontal plane.

A conical vessel with its base horizontal is filled partly with mercury and partly with water. Internally the radius of the base is 6 inches and the height 18 inches. The volume of the mercury is seven times that of the water. Find the pressure on the base, [illegible] that a cubic inch of water weighs [illegible] grains and a

4. Describe briefly the common hydrometer, and investigate a formula for comparing the specific gravities of fluids by means of it. A common hydrometer floating in a fluid whose specific gravity is 1·1 has 5 inches of the stem above the surface. When it floats in a liquid whose specific gravity is 1·2, its stem is 6 inches above the surface. How much of the stem will be above the surface when it floats in a liquid whose specific gravity is 1·3? (H)

5. Describe the hydraulic accumulator for storing up water under pressure. If the ram of the accumulator be 6 inches in diameter, what load will be required to produce a water pressure of 500 lb. on the square inch? To what head of water would this pressure correspond? (E)

XXXV. Science and Art Examinations. 1888.

1. Describe briefly the specific gravity bottle. *When the bottle is* full of water it is counterpoised by 983 grains in addition to the counterpoise of the empty bottle, and by 773 grains when filled with alcohol. What is the specific gravity of alcohol? (E)

2. A vessel is partly filled with water, and then oil is poured in it until it forms a layer 6 inches deep. Find the pressure per square inch due to the weight of the liquids at a point 8·5 inches below the upper surface of the oil, assuming the specific gravity of the oil to be 0·92, and the weight of a cubic inch of water to be 252 grs. (E)

3. If a vessel of any shape contains a liquid with a free surface, show that the pressure per square inch is constant for all points of the liquid situated in the same horizontal plane.

A sphere whose internal radius is 1 foot contains mercury which covers three-fourths of the vertical diameter. Find the resultant pressure on a horizontal plane passing through the centre of the sphere, assuming that a cubic inch of mercury weighs 3400 grns. (A)

4. Find a general expression for the depth of the centre of pressure of any plane area.

A triangular area has one of its angular points on the surface of the water, and its other two angular points at given depths (a and β) below the surface. Find the depth of the centre of pressure. (H)

5. If a cube whose specific gravity of 0·5 is placed in water wit four edges vertical, show that its position is one of unstable equil brium, and find the position in which it will come to rest. (H)*

6. The base of a rectangular tank for holding water is a squar 16 square feet in area. The sides of the tank are vertical, and i holds 250 gallons of water when quite full. Find the depth of th tank, and the pressures on each side and on the base when quit filled with water. (The weight of a gallon of water is 10 lb., and o a cubic foot of water $62\frac{1}{2}$ lb.) (E)

7. Sketch a vertical section through the cylinder and pump of hydrostatic press, showing the necessary valves and connexion Sketch separately the collar for packing the ram, and explain wh it is so efficient. How is the pressure taken off the object unde compression when required? Sketch the arrangement. What i the proportion of the diameters of the plunger and the ram whe the mechanical advantage gained thereby is 100 to 1, neglectin friction? (E)

XXXVI. Science and Art Examinations. 1890.

1. State the rule for finding the magnitude of the resultan pressure of a liquid on a plane area immersed in it.

A square, whose side is 8 feet long, has its plane immerse vertical and its upper edge on the surface of the water in which it i immersed. Find the magnitude of the resultant pressure on on face of it. If the square were fixed and the surface of the water wa raised a foot, what would now be the magnitude of the resultan pressure? (E)

2. State a method of finding the specific gravity of a liquid b means of a balance.

A glass ball weighs 1000 grains. It weighs 630 grains in wate and 650 grains in wine. Find the specific gravity of the wine. (E

3. Draw a diagram of a section of a hydraulic press, and explai the

When the ram is exerting a very great pressure, why may a comparatively weak pipe be strong enough to contain the water which transmits the force from the driving point? (E)

4. A cubical vessel is filled with two liquids whose specific gravities are 1 and 0·8. They do not mix, and their volumes are equal. Find the ratio of the resultant pressure on the upper to that on the lower half of one of the vertical faces of the cube. (A)

5. A volume (V) of air under a given pressure (P) is mixed with a volume (v) of air under a given pressure (p). The mixture occupies a given volume (U). Assuming that the temperatures of the constituents and of the mixture are the same, find the pressure of the mixture.

If 1000 cubic inches of air under a pressure of 20 lb. per square inch are mixed with 800 cubic inches of air under a pressure of 15 lb. per square inch, supposing the temperature the same, find the pressure of the mixture if it has a volume of 1500 cubic inches. (A)

6. A spherical shell made of a material of given specific gravity floats half immersed in water. Find the ratio of the internal to the external radius. (H)

XXXVII. Science and Art Examinations. 1891.

1. The glycerine barometer is found to stand at 329·2 inches when the mercurial barometer stands at 30·61 inches. Given that the specific gravity of mercury is 13·569, find the specific gravity of glycerine.

State the physical principles that justify your calculation. (E)

2. A cube, whose specific gravity is 0·75, floats in water. How much of one of its vertical edges is under water?

If its edges are 2 feet long, what is the magnitude of the resultant fluid pressure on the immersed part of one of its vertical faces? (E)

3. It may be assumed that 100 cubic inches of air weigh 33 grains when the barometer stands at 30 inches. Find the weight of the air that gets out of a room when the barometer falls from 30

inches to 29·5 inches, the dimensions of the room being 30 feet long, 20 feet wide, and 15 feet high.

State the physical principle that justifies your calculation. (E)

4. Two liquids that do not mix are placed in the same vessel. The density of the lower liquid is ρ and that of the other $m\rho$. A cylinder floats in them with its axis vertical, and is completely submerged; its density is $n\rho$. Find the condition that it may be half in the upper and half in the lower liquid. (A)

5. A hollow cone, whose vertical angle is a right angle, is held with its vertex downwards, and is filled with water. Find the magnitude and line of action of the resultant pressure on either of the portions into which it is divided by a vertical plane through its axis. (H)

MISCELLANEOUS EXPERIMENTAL EXERCISES.

1. Pour mercury into a U tube until it occupies about 2 inches of each arm. Pour water into one arm and methylated spirits into the other. Given that the specific gravity of mercury is 13·6, find the specific gravity of the spirit.

2. Pour mercury into a U tube until it occupies about 3 inches of each arm. Pour water into one arm (1st) about 4 inches deep, (2nd) 6 inches, (3rd) 8 inches, (4th) 10 inches, (5th) 12 inches, (6th) 14 inches; and prove that the ratio of the height of water to the height of mercury above the common level is constant.

3. A rectangular block of wood is given to you. Suspend a piece of lead from it by means of thread, and adjust the weight of the lead so that both float totally immersed in water. Prove that the weight of the lead and wood together is equal to the weight of the water they displace.

4. Fill a vessel with water to the brim, and weigh it. Now take the vessel out of the scale-pan and place a piece of wood in the water, and fill the vessel to the brim, and weigh it. Hence prove that the weight of the wood is equal to the weight of the water it displaces.

5. Pour water into a vessel until it is about half full. Float a piece of wood on the water, and weigh the vessel, water, and wood. Now force the wood down until it is totally immersed (this can be done by carefully pushing it down with the tips of three fingers). Find the additional weight required to produce a balance. Prove that this additional weight is equal to the additional weight of water displaced.

6. Find the magnitude of the force required to hold the given solid totally immersed in methylated spirits.

7. Prove experimentally that the force required to sink the given ball (the volume of which can be found by direct measurement) totally immersed in a liquid is equal to the difference between its weight and the weight of the liquid it displaces.

8. Take an iron cylinder (about 4 inches long and half-an-inch diameter), and make a net of cotton thread for it, so that it can be suspended with its axis vertical. Divide the length of the cylinder into 8 equal parts, and find its apparent weight (*a*) with one-eighth in a liquid, (*b*) two-eighths in the liquid, and so on; and prove that the force which the liquid exerts upon it is in each case equal to the weight of the liquid displaced. The volume of water displaced can be found by direct measurements.

9. The given solid is made up of lead and zinc. Given a piece of lead and a piece of zinc, find the proportion of lead and zinc in the solid.

10. Find, by using a mercury gauge, the height of the surface of water in the cistern.

11. Find the thickness of the given sheet of tinfoil.

12. A mixture of equal volumes of water and glycerine is given to you. Find the specific gravity of the glycerine.

13. Find the specific gravity of methylated spirits and water when mixed (*a*) in the ratio of 1 volume to 1 volume, (*b*) 1 volume to 2 volumes; (*c*) 1 volume to 3 volumes.

14. A saturated solution of copper sulphate is given to you. Find its specific gravity by weighing a large crystal of copper sulphate in it.

15. Given a Boyle's law tube, make three measurements to determine the height of the barometer.

TABLE I.

ENGLISH MEASURES OF LENGTH, AREA, AND VOLUME.

LENGTH.

Standard ... ... The **Yard.**

12 inches (″)	=	1 foot (′).
3 feet	=	1 yard.
1760 yards	=	1 mile.
2029 yards	=	1 knot.
100 links	=	1 chain.
22 yards	=	1 chain.

AREA.

144 sq. inches	=	1 sq. foot.
9 sq. ft.	=	1 sq. yd.
4840 sq. yds.	=	1 acre.
640 acres	=	1 sq. mile.

VOLUME.

1728 cubic inches	=	1 cub. foot.
27 cub. ft.	=	1 cub. yd.
277·274 cub. inches	=	1 gallon.

A gallon is the volume occupied by 10 lb. of water at 4° C.

TABLE IA.

METRIC MEASURES OF LENGTH, AREA, AND VOLUME.

LENGTH.

Standard ... ... The **Metre.**

10 metre (m.)	=	1 decametre.
100 m.	=	1 hectometre.
1000 m.	=	1 kilometre.
10 decimetre	=	1 metre.
100 centimetre (cm.)	=	1 metre.
1000 millimetre (mm.)	=	1 metre.

AREA.

100 sq. m. = 1 sq. decametre.
10,000 sq. m. = 1 sq. hectometre = 1 hectare.
1,000,000 sq. m. = 1 sq. kilometre.
100 sq. decimetre = 1 sq. m.
10,000 sq. cm. = 1 sq. m.
1,000,000 sq. mm. = 1 sq. m.

VOLUME.

1000 cubic centimetre (cc.) = 1 litre (cubic decimetre).
1 itre = 1 stere (1 cubic metre).

TABLE II.

RELATION BETWEEN THE ENGLISH AND METRIC UNITS.

LENGTH.

1 metre = 39·3704 inches = 3·2808.
1 foot = 30·48 cm.
1 mile = 1·6093 kilom.
1 kilom. = ·6214 mile.

AREA.

1 sq. inch = 6·451 sq. cm.
1 sq. foot = 929 sq. cm.
1 acre = 4046·7 sq. m.
1 sq. cm. = ·155 sq. m.
1 sq. m. = 1·196 sq. yd.
1 hectare = 2·471 acres.

VOLUME.

1 cub. inch = 16·386 cc.
1 cub. foot = 28·315 litres.
1 cc. = ·06103 cub. in.
1 litre = 61·027 cub. in.
1 litre = ·2201 gals.

TABLE III.

AREAS AND VOLUMES.

($\pi = 3.1416$ or $3\frac{1}{7}$ nearly.)

ea of a triangle is half the area of the rectangle on the same base and of the same vertical height.

2. Area of a circle is π times the area of the square on the radius, *i.e.* it is nearly $3\frac{1}{7}$ times the area of the square on the radius.

3. Volume of a circular cylinder, the radius of the base of which is r units of length and height h units, is $\pi r^2 h$ units of volume.

4. Volume of a pyramid is one-third the volume of a prism on the same base and of the same height.

5. Volume of a cone is one-third the volume of a cylinder on the same base and of the same height.

6. Area of the surface of a sphere of radius r is $4\pi r^2$.

7. Volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

TABLE IV.

ENGLISH MEASURES OF MASS.

Standard ... ... Pound Avoirdupois.

437·5 grains (gr.)	= 1 ounce Avoirdupois (oz.)
7000 grs.	= 16 oz. = 1 pound (lb.)
112 lb.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton.

1 ton of water 224 gallons or 35·9 cubic feet.

METRIC MEASURES OF MASS.

Standard ... ... Kilogramme.

10 gramme (gm.)	= 1 decagramme.
100 gm.	= 1 hectogramme.
1000 gm.	= 1 kilogramme (kilogm.)
10 decigm.	= 1 gm.
100 centigm.	= 1 gm.
1000 milligm.	= 1 gm.
1000 kilogm.	= 1 tonne.

TABLE V.

RELATION BETWEEN THE ENGLISH AND METRIC MEASURES OF MASS.

1 kilogramme	= 15432·349 grains.
1 grain	= ·0648 gm.
1 oz.	= 28·35 gm.
1 lb.	= 453·593 gm.
1 cwt.	= 50·8 kilogm.
1 ton	= 1016·05 kilogm.
1 gm.	= 15·432 grains.
1 kilogm.	= 2·2046 lb.
1 tonne	= ·9842 ton.

TABLE VI.

SPECIFIC GRAVITIES OF SUBSTANCES.

Aluminium	2·7	Steel	7·76
Ashwood	·753	Iron (cast)	7·2
Boxwood	·96	Iron (wrought)	7·76
Brass	8·00	Ivory	1·92
Carbon	1·8	Lead	11·4
Copper	8·95	Mahogany	·56 to ·85
Cork	·24	Mercury	13·596
Diamond	3·5	Oak	·69 to ·99
Ebony	1·19	Platinum	21·5
Glass (crown)	2·5	Sea-water	1·026
Glass (flint)	3 to 3·6	Silver	10·57
Glycerine	1·26	Sulphur	2·07
Gold	19·3	Tin	7·3
Guttapercha	·97	Turpentine	·87
Ice	·918	Wax (bees)	·96
India rubber	·99	Zinc	7·2

ANSWERS.

NOTE.—Except where otherwise stated, the following values have been taken: Specific gravity of iron, 7·5; copper, 8·8; mercury, 13·5; gold, 19·3; lead, 11·3; silver, 10·4; glycerine, 1·25; alcohol, ·85; zinc, 7·1; platinum, 21·5; air at S.T.P. ·0013; ice, ·9; wood, ·5; glass, 2·5; cork, ·25; also 1 foot = 30·48 cm.; and $\pi = 3\frac{1}{7}$.

CHAPTER I.

p. 10.

2. 594·36 cm.; 16·82496 kilom.; 4,870,150 cc.; 762 mm.; 975·36 cm.

3. 29·921 in.; 282 yds. 2 ft. 1·165 in.; 183·07 ... cub. in.; 14·126 cub. ft.; 32·18 ft.

CHAPTER II.

p. 18.

1. (*a*) 7·5; (*b*) ·4996; (*c*) 11·31; (*d*) 10·3; (*e*) ·79; (*f*) 20·998.

2. (*a*) 1875 lbwt.; (*b*) 74250 lbwt.; (*c*) 62·47 ozwt.; (*d*) 8475 gmwt.; (*e*) 57900 kilo-gmwt.; (*f*) 818750 gmwt.

3. (*a*) 1·463 cub. ft.; (*b*) ·249 cub. ft.; (*c*) 23·89 cub. ft.; (*d*) 98·352 cc.; (*e*) ·000588 cc.; (*f*) 150,000 cc.

p. 23.

1. 1. **2.** ·875. **3.** [illegible]. **4.** 8·8 gmwt.

5. 57·34 inches.

pp. 34—39.

1. (*a*) $133\cdot\dot{3}$ cc.; (*b*) 65077·72 cc.; (*c*) 37·407 cc.; (*d*) 2·4107 cu ft.; (*e*) 215·04 cub. ft.; (*f*) 10·06 cub. ft.; (*g*) 9·846 cub. ft

2. (*a*) 7500 gmwt.; (*b*) 30243·1 gmwt.; (*c*) 6826·545 gmwt 734531·25 lbwt.; (*d*) 397800 gmwt.; (*e*) 36041·6 ozwt.

3. (*a*) ·85; (*b*) 1·845; (*c*) 8·8; (*d*) 7·5; (*e*) ·746; (*f*) ·00129.

4. 1022 lbwt. **5.** 2667 ozwt. **6.** 5156·25 lbwt.

7. 9·09 sq. cm.; 3·401 cm. **8.** 375. **9.** 7366·07 lbw

10. ·1202 cm. **11.** ·055 cm. **12.** 2.36.

13. 147·09 lbwt. **14.** 10·22 lbwt. **15.** 135 lbwt.
16. ·905. **17.** ·8. **18.** 8·109 lbwt.; ·470 lbwt.
19. 5·440 gmwt.; 73·44 gmwt. **20.** 8·59 cub. in.
21. ·0095 cub. ft. **22.** 300 cc. **23.** 5·66 ozwt.; 134·4 cub. in.
24. 9 cub. in. **25.** 1,290,000 gmwt.; 2·48076 × 10^6 litres.
26. 7·8125 ozwt.; 5787 ozwt.; ·7233 ozwt.; ·4918 ozwt.
27. 7071·429 gmwt.; 645·76 gmwt.; 445·24 gmwt.
28. 32·74 ozwt. **29.** 13500 gmwt.
30. (*a*) ·00116 lb.; (*b*) ·00497 lb.; (*c*) ·01123 lb.
31. ·0638 lb. **32.** 637,520 gm. **33.** ·00559.
34. 11173·2 cc. **35.** 69·755 litre. **36.** 8257·54 cub. in.
37. $\frac{5}{8}$. **38.** $1\frac{1}{3}$ ozwt. **39.** 8·5. **40.** ·36 cub. ft.; $\frac{1}{18376}$.
41. 7·031. **42.** 177975 lbwt. **43.** 33·99 ft.
44. 4; 3·75. **45.** 17·587. **46.** 1·42. **47.** 6·25.

CHAPTER III.

pp. 43—44.

1. (*a*) 1; (*b*) ·068; (*c*) 20·74; (*d*) 14·7.
2. (*a*) 186·66; (*b*) 12·47; (*c*) 36·66; (*d*) 102·6.
3. (*a*) 8325 lbwt.; (*b*) 2,230,272,000 lbwt.; (*c*) 56000 gmwt.

p. 55.

1. 43·402 lbwt per sq. in.; 14·647 lbwt. per sq. in.
2. 9765·625 lbwt. **3.** 140·95 in. **4.** 63058·5 gmwt.
5. 30·72 in. **6.** 150,000 lbwt.

pp. 68—73.

1. 1·085; 14·647; ·924. **2.** 130·208. **3.** ·4882; ·4340.
4. 406·4256 in. **5.** 30·1056 in.
6. (*a*) 62·5; (*b*) ·434; (*c*) ·0294; (*d*) $\frac{3}{8}$.
7. (*a*) ·192 in.; 27·648 in.; (*b*) ·0142 in; (*c*) 2·0448 in.
8. 1026; 13·5. **9.** 403·93. **10.** 67324·5.
11. 1,013,800. **12.** 14·84. **13.** 45.
14. 19225·45 lbwt. **15.** 856·27 lbwt. **16.** 30937·5 lb.
17. 3142·857 lbwt. **18.** $\frac{1}{33}$ of length of one arm.

19. The water occupies 6·0357 in., and the alcohol 5·9643 in.

20. 9·23 in. **22.** 5·83 lbwt. per sq. in.

23. (*a*) 271·4 lbwt. per sq. in.; (*b*) 351734·4 lbwt. per sq. yd.

24. $\frac{1}{12}$ lbwt. per sq. in. **25.** 8437·5 lbwt. per sq. ft.

26. 13·504; 2160 lbwt. per sq. ft. **27.** 2291·6 lbwt. per sq. in.

28. 12·586 lbwt. per sq. in. **29.** 98 ft. **30.** 1 : 2 nearly.

31. 14 lbwt. per sq. in. **32.** 25 lb. 1·9 ozwt. per sq. in.

35. Pressure is 2·893 ozwt. per sq. in.; force on the base 81·018 ozwt.; force on the table 114·024 ozwt.

36. 36·864 ft. **37.** 3 : 2. **38.** 1250 lbwt.; 1312·5 lbwt.

39. The common surface is at the bottom of one branch.

40. 31·25 lbwt. **41.** 93·75 lbwt. **42.** 9 : 5.

43. 18 : 35; 3 : 5. **44.** 4½ inches. **47.** 8 : 5 : 3.

CHAPTER IV.

pp. 86—87.

1. (*a*) 375 lbwt., 2812·5 lbwt.; (*b*) 375 lbwt., 187·5 lbwt.; (*c*) 36·16 lbwt., 759·36 lbwt.; (*d*) 205 gmwt., 51·125 gmwt.; (*e*) 203·3625 gmwt., 101·68125 gmwt.

2. (*a*) 5062·5 lbwt.; (*b*) 5062·5 lbwt.; (*c*) 488·16 lbwt.; (*d*) 2767·5 gmwt.; (*e*) 2745·39 gmwt.

3. (*a*) 318·75 lbwt.; (*b*) 318·75 lbwt.; (*c*) 30·736 lbwt.; (*d*) 174·25 gmwt.; (*e*) 172·858 gmwt.

4. (*a*) 2437·5 lbwt. upwards; (*b*) 187·5 lbwt. downwards; (*c*) 723·2 lbwt. upwards; (*d*) 153·875 gmwt. downwards; (*e*) 101·68125 gmwt. downwards.

5. (*a*) 2250 lbwt. downwards; (*b*) 4875 lbwt. downwards; (*c*) 271·2 lbwt. upwards; (*d*) 2716·375 gmwt. downwards; (*e*) 2643·70875 gmwt. downwards.

6. (*a*) 2493·75 lbwt. upwards; (*b*) 131·25 lbwt. downwards; (*c*) 728·624 lbwt. upwards; (*d*) 123·125 gmwt. downwards; (*e*) 71·1[illegible]875 gmwt. downwards.

7. 55·555 cc.; 55·555 cc.

8. 750 gmwt.; 750 gmwt.; 250 gmwt.

9. 113 gmwt.; 8·5 gmwt.; 104·5 gmwt. **10.** 4 lbwt.; 8·4 lbwt.

11. 27 gmwt.; 27 cc.; 7·592 gm. per cc.; 7·592.

12. 20 gmwt.; 27·33 cc.; ·731 gmwt.

pp. 113—123.

1. (*a*) 25 gmwt.; (*b*) 781·25 lbwt.; (*c*) 33750 lbwt.; (*d*) 2·428 lbwt.; (*e*) 425 gmwt.
2. (*a*) 375 gmwt., sinks; (*b*) 781·25 lbwt., floats; (*c*) 33000 lbwt., floats; (*d*) 25·2 lbwt., sinks; (*e*) 5650 gmwt., sinks.
3. 25 gmwt.; 25 cc.; 75 cc.
4. 202·5 gmwt.; 15 cc.; 15 cc.
5. 3·6 gmwt.; 23·4 gmwt.; 4·5 gmwt.; 22·5 gmwt.
6. 55·49 lbwt.
7. ·25.
8. 11·3̇6̇ lbwt.; 14·2 lbwt.; 88·6̇3̇ lbwt.; 85·8 lbwt.
9. 86·54 gmwt.
10. 150 gmwt.; no force. The body will just float totally immersed in alcohol.
11. 975 gmwt.; 47·56 cc.
12. 3·616 lbwt.; 7·232 lbwt.; ·702 lbwt.
13. 1·19 ft. nearly.
14. 34·722 lbwt.
15. 500 cc.
16. 23·1̇8̇ gmwt.; 29·268 gmwt.
17. 1·5.
18. 1·77 gmwt.
19. $\frac{7}{25}$.
20. 2·78 cm.
21. 3089·82 gmwt.
22. 2852·14 gmwt.
23. 1074·22 lbwt.; 952·03 lbwt.
24. 4·42 gmwt; 2·651 gmwt.
25. 26·47 cc.; 7·5.
26. $\frac{78}{81}$.
27. (*a*) half the weight of the solid; (*b*) $\frac{15}{26}$ weight of solid.
28. $16\frac{182}{316}$ ozwt. or 1·05 lbwt.
29. 843·75 lbwt.
30. 281·25 lbwt.
31. $2\frac{2}{9}$ cub. in.
32. 13·5.
33. 2 cub. ft. 971·68 cub. in.
34. $7\frac{1}{15}$ lbwt.; 56 lbwt.
35. 2·4 cub. ft.
36. 2·4 feet edge.
37. 8; 34·56 cub. in.
38. 6.
39. $252\frac{21}{32}$ grswt.
40. 4 : 19.
41. $62\frac{4}{11}$ grswt.
42. ·858; ·7947 ozwt.
43. 2·5.
44. ·865.
45. 1·841.
46. ·25.
47. 8·5; ·85.
49. 111·105 gmwt.
50. $\frac{25}{38}$.
51. $3\frac{1}{3}$.
52. 481 : 4559 or ·1055 : 1.
53. Two-thirds of its area immersed.
57. The weight the string has to bear is 12 lbwt. The change in the pressure at any point of the bottom of the bucket depends upon the increase in the depth of the water. The change in the force on the bottom depends upon the increase in the depth of the water and on the area of the bottom. The change in the force on the floor is 2 lbwt.
58. Two-thirds in mercury; one-third in oil.

62. 18 cubic inches in each.
64. $\frac{193}{253}$ of its volume in mercury; $\frac{60}{253}$ in oil.
65. $5\frac{14}{25}$ lbwt. in water; $6\frac{6}{25}$ lbwt. in mercury. 66. $4\frac{3}{4}$ in.
67. 1 cub. inch. 69. $\frac{8}{9}$. 72. 2·5. 73. 1·728.
74. 10 : 13. 75. $2\frac{3}{7}$ ozwt. 76. 8·903. 77. ·965.
78. 1·985. 79. 1·077. 80. 11 : 9.
81. 387·761 grawt. 82. ·8. 83. ·041 ft. $[\sqrt[3]{\frac{44}{39}} - 1]$.
84. 144 : 73. 85. 2·23 inches.
86. $\frac{am + bn}{a + b}$ th part. 88. 8.
89. 9·011 cubic inches. 90. ·947.

If the highest sp. gr. given by a common hydrometer is s, and the lowest is s', and the distance between these graduations is l, and the volume up to the lowest division is V, then it can be proved that $Vs = (V + al)s'$ (since each is equal to weight of hydrometer). If also σ is the sp. gr. denoted by the division, the distance of which from the lowest graduation is x, $Vs = (V + ax)\sigma$. Hence, eliminating V, we get $\frac{s - \sigma}{s - s'} = \frac{x \cdot \sigma}{ls'}$.

91. $\frac{40}{41}$; $\frac{40}{42}$; $\frac{40}{43}$; $\frac{40}{44}$; $\frac{40}{45}$; $\frac{40}{46}$; $\frac{40}{47}$; $\frac{40}{48}$; $\frac{40}{49}$.
[If x is the number of a division from the lowest, then $s(x + 40) = 40$].
92. 1·280; 1·250; 1·221; 1·196; 1·167; 1·143; 1·119; 1·096; 1·074; 1·052; 1·032.

For the xth division $s = \frac{20}{15 + \frac{1}{8} + \frac{3x}{8}} = \frac{160}{125 + 3x}$.

CHAPTER V.

pp. 132—133.

1. 25·8789 lbwt. per sq. in.
2. 29·33 inches; 26·4 ft.; 1·35 inches; 1·08 inches.
3. 14·92 lbwt. per sq. in.
4. 14·6 inches; 370·84 mm.; 7·128 lbwt. per sq. in.; 500·63 grwt per sq. cm.
5. 28·67 inches. 6. 156·25 lbwt. 7. 13·5 inches.

pp. 141—143.

1. $2\frac{1}{3}$ lbwt. per sq. in. **2.** $666\frac{2}{3}$ lbwt. per sq. in.

3. *70 cub. ft.* **4.** *16·78 cc.* **5.** *12·23125 cc.*

6. $62\frac{2}{3}$ lbwt. per sq. in. **7.** $1088\frac{1}{3}$ mm. of mercury.

8. 13060 cc. **9.** 329·63; 270·59; 238·55; 217·47; 169·97 mm.

10. 7·125 in. **11.** ·5074. **12.** 45·389 cc.

13. 422·727 cc. **14.** 1·35 gmwt.; ·136 cc.; 1·836 gmwt.

15. 99·871 gmwt.; $33\frac{1}{3}$ cc.; $33\frac{1}{3}$ gmwt. It will sink, because the force upon it upwards is less than its weight.

pp. 146—147.

1. ·0044785 gm. **2.** 5·6099 gm. **3.** ·2836 gm.

4. ·00006789 gm. **5.** 6626·25 cc. **6.** 296856 cc.

7. 7781 cc. **8.** 589·14 mm. of mercury.

9. 404·65 mm. of mercury. **10.** 247·41 mm. of mercury.

11. 450·5 cc. **12.** ·3394 gm. **13.** 471·3 cc.

pp. 153—154.

1. The surface of the mercury in the vessel must be at a depth of 3·27 inches.

2. The top of the bell is 15·25 ft. below the surface.

3. ·001433; 49·9928. (Observe that inside the bell the wood must rise out of the water. *See* Note 5, p. 151).

4. 3·226 ft. **5.** 9·985 in. **6.** 1818·18 cub. ft.

p. 156.

650 ozwt; 487·5 ozwt.

pp. 165—173.

1. ·26 gmwt. (Note how sensible the difference is.)

2. 27·0047 gmwt.; 27·1411 gmwt. (Comparing these shows how much more the air affects the true weight of cork than does iron.)

3. 199·9675 gmwt. **4.** 774·18 mm. **5.** 4·395.

6. 13·5 inches. **7.** 26·62 inches.

8. 10 mm.; 13243·5 dynes per sq. cm.

9. 661500 dynes per sq. cm.

10. (*a*) 760 cc.; (*b*) 9·75 cc.; (*c*) 168 cub. in.

(a) 9 lbwt. per sq. in., 15·97 lbwt. per sq. in.; (b) 6·428 lbwt. per sq. in., 17·07 lbwt. per sq. in.; (c) 5 lbwt. per sq. in., 18·3 lbwt. per sq. in.

12. 380 mm.; 506·6 mm.; 542·86 mm.; 581·18 mm.

13. 30·53 inches. 14. ·01 cub. in.

16. ·466 cub. inches (supposing area of section is 1 sq. in.).

17. 755·1 mm. nearly. 18. 276·48 inches.

19. $\frac{1000}{197}$ lbwt. per sq. in. 20. $\frac{7600}{18}$ mm. of mercury.

21. ·0135 gm. 22. 11359 cc. 23. 26·8 lbwt. per sq. in.

24. 2·929 lbwt. per sq. in.; 6 inches. 25. 636 mm.

26. 5 cm. 27. 48·82 lbwt. per sq. in.

28. 16·125; 13·03; 10·655; 8·86; 8·365; 7·9125 inches.

[If x is the length of the tube occupied by the air when the pressure is equal to h inches of mercury, then (the tube having uniform bore) x is given by the equation $2x^2 + x(h-40) - 600 = 0$]. It will be seen that equal divisions do not correspond to equal changes of pressure. This question shows how to graduate a closed tube pressure gauge.

29. $\frac{15}{32}$ inch. 30. 2 cubic inches. 31. 4 feet.

34. The surface of the water within the bell is at a depth of 238 ft.

35. $13\frac{44}{64}$ lbwt. per sq. in. 36. $37\frac{1}{3}$ lbwt. per sq. in.

37. $22\frac{181}{288}$ lbwt. per sq. in. 38. 7·932 ozwt.; 3·151 cub. feet.

39. The specific gravity of air at $14\frac{3}{4}$ lbwt. per sq. in. is ·0013, hence the pressure in the vessel is $1\frac{73}{104}$ lbwt. per sq. in.

40. $31\frac{1}{59}$ cubic feet.

41. $\frac{a+5}{6}$ cubic inches, where a is the length of the Torricellian vacuum.

42. $7\frac{1}{3}$ cubic inches (the barometer standing at 30 inches).

43. $3\frac{5}{6}$ cubic inches. 46. 33 ft. $2\frac{1}{4}$ in. 47. 1·255.

48. ·99956 cubic feet. 53. 14·214. 54. ·0000$\dot{7}$ cub. in.

55. 79 : 45. 56. 3 tons $917\frac{1}{7}$ lbwt.

57. 10336 mm.; 8203·174 mm. 58. 1,580,000 gmwt.

59. 120 lbwt. per sq. inch. 60. ·000533.

61. 124·196 gmwt. 62. $1\frac{1}{10}$ cubic inch.

63. 3 inches; 32 feet. 64. $21\frac{3}{7}$ lbwt. per sq. inch.

66. 124 ozwt.; ·34 oz. per cub. ft.; 30394 feet.

CHAPTER VI.

pp. 183—184.

1. 120·04 lbwt. **2.** 1657·3 lbwt. **3.** 221·354 lbwt.

4. 3268·571 lbwt.; 149·76 ft. (pressure of atmosphere being 15 lbwt. per sq. inch). **5.** 33 ft. 5·4 in.

pp. 193—194.

1. $\frac{10000}{14641}$ *of standard pressure*; *·00000501 oz.*

2. ·8642 gm. per litre. **3.** Between the 3rd and 4th.

4. Between the 8th and 9th. **5.** 6 gm.; 4560 mm. of mercury.

p. 197.

1. 25900 lbwt. **2.** $3\frac{5}{8}$ tonwt.

3. 1·4617 inches; 37·7538 inches. **4.** ·59696 foot.

pp. 197—201.

1. 436·508 lbwt. **2.** 511·532 lbwt.; 2046·128 lbwt.

3. 188·568 lbwt. **4.** 2·97 inches.

5. 1st stroke, 8·766 ft.; 2nd stroke, 15·19 ft.; 3rd strok the vessel; at end of 1st stroke, 730·5 lbwt.; at en stroke, 1265·83 lbwt.

6. 1257·142 cubic feet; 78571·375 lb. **7.** 2 feet.

10. 5. **11.** After the 10th. **12.** 4·35 lbwt. per sq. inch.

13. 1 ton 260 lbwt. **15.** 6th. **16.** 33 ft. 4 in.

18. 8. **20.** $6\frac{2}{7}$ feet; 6·32 feet; 3 strokes.

21. 10·10 inches. **22.** 10·08 inches. **23.** 6·8 feet nearly.

24. 1·097 tonwt. **25.** 17·875 tonwt. **26.** 1·364 feet.

27. 7·77 lbwt.

CHAPTER VII.

pp. 213—214.

1. (*a*) 250 lbwt.; (*b*) ·204 lbwt.; (*c*) 5000 lbwt.; (*d*) 43·65 lbwt.

2. $1078933\frac{1}{3}$ gmwt.; $38533\frac{1}{3}$ gmwt.

3. 36·169 lbwt.; 18·0845 lbwt. **4.** 3750 lbwt.

5. 15625 lbwt.; 15625 lbwt. **6.** 62779 tonwt.

7. 1125 lbwt.; 2·405 ft. If the side is *a*, then the force on the hexagon is $\frac{9a^3}{4}w$; and depth of c.p. $= \frac{25\sqrt{3}}{36}$.

8. (*a*) On the top 69·44 lbwt., on the bottom 87.96 lbwt., on the sides 78·70 lbwt.; (*b*) 916·66 lbwt. addititional force.